

PROBLEMES D'APLICACIÓ DE LA REGLA DE L'HÔPITAL.

a) $\lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{e^x - x - 1}$

b) $\lim_{x \rightarrow 0} \frac{6 \sin x - 6x + x^3}{6x^5}$

c) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\operatorname{tg} 3x}$

d) $\lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^2 \sin x^2}$

e) $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$

f) $\lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{x - \sin x}$

g) $\lim_{x \rightarrow \infty} \frac{x^4 - x^2}{e^x + 1}$

h) $\lim_{x \rightarrow 0} x^2 \ln x$

i) $\lim_{x \rightarrow \frac{\pi}{2}} (\sin x - \cos x)^{\operatorname{tg} x}$

j) $\lim_{x \rightarrow \infty} x \cdot \sin \frac{a}{x}$

k) $\lim_{x \rightarrow 1} \left(\frac{x}{\ln x} - \frac{1}{\ln x} \right)$

l) $\lim_{x \rightarrow 1} x^{1-x}$

m) $\lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x(1 - \cos x)}$

n) $\lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}}$

p) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(\sin x)}{(\pi - 2x)^2}$

q) $\lim_{x \rightarrow 0} x^{7x}$

Solucions

$$a) \lim_{x \rightarrow 0} \frac{\ln(x^2 + 1)}{e^x - x - 1} = \lim_{x \rightarrow 0} \frac{\frac{2x}{x^2 + 1}}{\frac{0}{0}} = \lim_{x \rightarrow 0} \frac{2x}{(x^2 + 1)(e^x - 1)} = \lim_{x \rightarrow 0} \frac{2}{2xe^x + (x^2 + 1)e^x} = 2.$$

$$b) \lim_{x \rightarrow 0} \frac{6 \sin x - 6x + x^3}{6x^5} = \lim_{x \rightarrow 0} \frac{6 \cos x - 6 + 3x^2}{30x^4} = \lim_{x \rightarrow 0} \frac{-6 \sin x + 6x}{120x^3} = \lim_{x \rightarrow 0} \frac{-6 \cos x + 6}{360x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{6 \sin x}{720x} = \frac{6}{720} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{6}{720} \cdot 1 = \frac{1}{120}$$

$$c) \lim_{x \rightarrow 0} \frac{\operatorname{tg} 2x}{\operatorname{tg} 3x} = \lim_{x \rightarrow 0} \frac{\frac{2}{\cos^2 2x}}{\frac{3}{\cos^2 3x}} = \lim_{x \rightarrow 0} \frac{2 \cos^2 3x}{3 \cos^2 2x} = \frac{2}{3}$$

$$d) \lim_{x \rightarrow 0} \frac{1 - \cos x^2}{x^2 \sin x^2} = \lim_{x \rightarrow 0} \frac{\sin x^2 \cdot 2x}{2x \sin x^2 + 2x^3 \cos x^2} = \lim_{x \rightarrow 0} \frac{\sin x^2}{\sin x^2 + x^2 \cos x^2} =$$

$$= \lim_{x \rightarrow 0} \frac{2x \cos x^2}{2x \cos x^2 + 2x \cos x^2 - 2x^3 \sin x^2} = \lim_{x \rightarrow 0} \frac{\cos x^2}{2 \cos x^2 + x^2 \sin x^2} = \frac{1}{2}$$

$$e) \lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx} = \lim_{x \rightarrow 0} \frac{a \cos ax}{b \cos bx} = \frac{a}{b}$$

$$f) \lim_{x \rightarrow 0} \frac{\operatorname{tg} x - x}{x - \sin x} = \lim_{x \rightarrow 0} \frac{1 + \operatorname{tg}^2 x - 1}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{\operatorname{tg}^2 x}{1 - \cos x} = \lim_{x \rightarrow 0} \frac{2 \operatorname{tg} x (1 + \operatorname{tg}^2 x)}{\sin x} = \lim_{x \rightarrow 0} \frac{2(1 + \operatorname{tg}^2 x)}{\cos x} = 2$$

$$g) \lim_{x \rightarrow \infty} \frac{x^4 - x^2}{e^x + 1} = \lim_{x \rightarrow \infty} \frac{4x^3 - 2x}{e^x} = \lim_{x \rightarrow \infty} \frac{12x^2 - 2}{e^x} = \lim_{x \rightarrow \infty} \frac{24x}{e^x} = \lim_{x \rightarrow \infty} \frac{24}{e^x} = 0$$

$$h) \lim_{x \rightarrow 0} x^2 \ln x = \lim_{x \rightarrow 0} \frac{\ln x}{\frac{1}{x^2}} = \lim_{x \rightarrow 0} \frac{\frac{1}{x}}{-2} = \lim_{x \rightarrow 0} \frac{x^2}{-2} = 0$$

$$i) \lim_{x \rightarrow \frac{\pi}{2}} (\sin x - \cos x)^{\operatorname{tg} x} = \lambda$$

$$\ln \lambda = \lim_{x \rightarrow \frac{\pi}{2}} \ln \left((\sin x - \cos x)^{\operatorname{tg} x} \right) = \lim_{x \rightarrow \frac{\pi}{2}} \operatorname{tg} x (\ln(\sin x - \cos x)) = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\sin x \cdot \ln(\sin x - \cos x)}{\cos x} =$$

$$= \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x \cdot \ln(\sin x - \cos x) + \sin x \frac{\cos x + \sin x}{\sin x - \cos x}}{-\sin x} = -1, \quad \ln \lambda = -1 \Rightarrow \lambda = e^{-1}$$

$$j) \lim_{x \rightarrow \infty} x \cdot \sin \frac{a}{x} = \lim_{x \rightarrow \infty} \frac{\sin \frac{a}{x}}{\frac{1}{x}} = \lim_{\substack{0_H \\ 0}} \frac{\cos \frac{a}{x} \cdot \frac{-a}{x^2}}{\frac{-1}{x^2}} = \lim_{x \rightarrow \infty} a \cdot \cos \frac{a}{x} = a$$

$$k) \lim_{x \rightarrow 1} \left(\frac{x}{\ln x} - \frac{1}{\ln x} \right) = \lim_{x \rightarrow 1} \frac{x-1}{\ln x} = \lim_{\substack{0_H \\ 0}} \frac{1}{\frac{1}{x}} = \lim_{x \rightarrow 1} x = 1$$

$$l) \lambda = \lim_{x \rightarrow 1} x^{1-x} = 1^\infty$$

$$\ln \lambda = \lim_{x \rightarrow 1} \ln \left(x^{1-x} \right) = \lim_{x \rightarrow 1} \frac{\ln x}{1-x} = \lim_{\substack{0_H \\ 0}} \frac{\frac{1}{x}}{-1} = -1 \ln \lambda = -1 \Rightarrow \lambda = e^{-1}$$

$$m) \lim_{x \rightarrow 0} \frac{\sin x - x \cos x}{x(1 - \cos x)} = \lim_{\substack{0_H \\ 0}} \frac{\cos x - \cos x + x \sin x}{1 - \cos x + x \sin x} = \lim_{x \rightarrow 0} \frac{x \sin x}{1 - \cos x + x \sin x} =$$

$$= \lim_{\substack{0_H \\ 0}} \frac{\sin x + x \cos x}{2 \sin x + x \cos x} = \lim_{\substack{0_H \\ 0}} \frac{2 \cos x - x \sin x}{3 \cos x - x \sin x} = \frac{2}{3}$$

$$n) \lambda = \lim_{x \rightarrow 0} \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}} = 1^\infty$$

$$\ln \lambda = \lim_{x \rightarrow 0} \ln \left(\frac{a^x + b^x}{2} \right)^{\frac{1}{x}} = \lim_{x \rightarrow 0} \frac{\ln \left(\frac{a^x + b^x}{2} \right)}{x} = \lim_{\substack{0_H \\ 0}} \frac{\frac{2}{a^x + b^x} \cdot \frac{1}{2} (a^x \ln a + b^x \ln b)}{1} =$$

$$= \frac{1}{2} (\ln a + \ln b) = \ln(ab)^{1/2} \ln \lambda = \ln(ab)^{1/2} \Rightarrow \lambda = \sqrt{ab}$$

$$p) \lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(\sin x)}{(\pi - 2x)^2} = \lim_{\substack{0_H \\ 0}} \frac{\frac{\cos x}{\sin x}}{2(\pi - 2x)^2} = \lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{4(\pi - 2x) \sin x} =$$

$$= \lim_{\substack{0_H \\ 0}} \frac{-\sin x}{4(-2 \sin x + (\pi - 2x) \cos x)} = \frac{-1}{4(-2+0)} = \frac{1}{8}$$

$$q) \lambda = \lim_{x \rightarrow 0} x^{7x} = 0^0$$

$$\ln \lambda = \lim_{x \rightarrow 0} \ln(x^{7x}) = \lim_{x \rightarrow 0} 7x \ln x = \lim_{\substack{0_H \\ \infty}} \frac{\ln x}{\frac{1}{7x}} = \lim_{\substack{\infty_H \\ \infty}} \frac{\frac{1}{x}}{\frac{-1}{7x^2}} = \lim_{x \rightarrow 0} -7x = 0$$

$$\ln \lambda = 0 \Rightarrow \lambda = e^0 = 1$$