

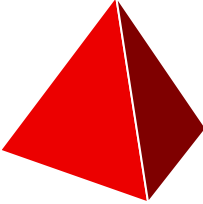
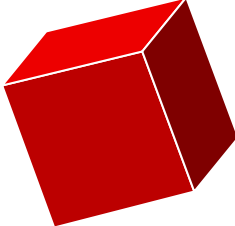
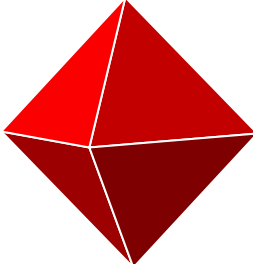
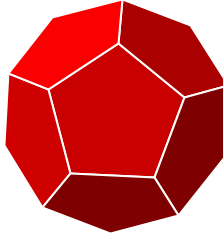
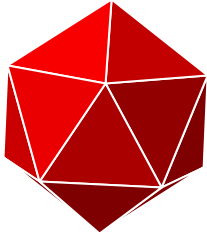
Políedros

Políedres

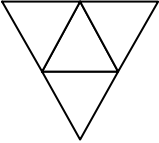
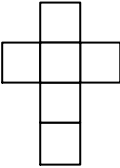
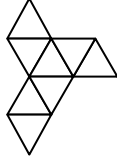
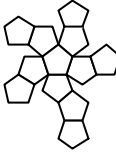

Teorema d'Euler
Políedres regulars
Políedres arquimedians
Sòlids de Catalan
Políedres duals
Prismes i antiprismes
Dipiràmides i deltàedres
Simetries del cub
Simetries de l'octàedre
Empaquetaments de políedres
Mesures dels políedres regulars i arquimedians
La bresca de mel.

Políedres regulars o platònics

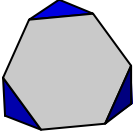
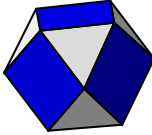
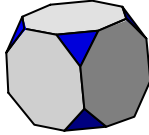
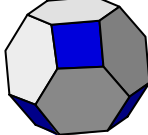
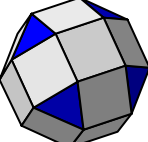

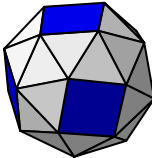
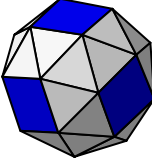
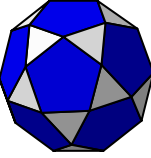
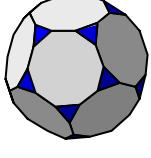
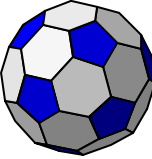
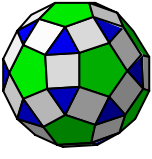
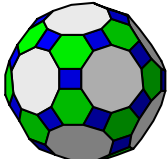
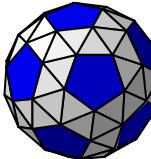
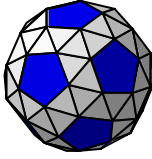
Políedres regulars o platònics són els políedres convexes tal que les seues cares són polígons regulars i cadascun dels vèrtexs el formen el mateix nombre de cares (ordre del vèrtex). Hi ha només 5 políedres regulars:

| | | | | |
|---|---|---|---|---|
|  |  |  |  |  |
| Tetràedre | Cub | Octàedre | Dodecàedre | Icosàedre |

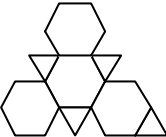
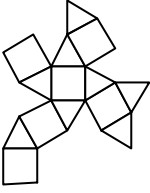
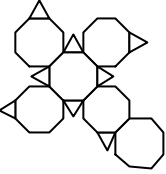
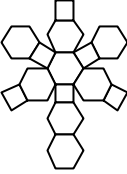
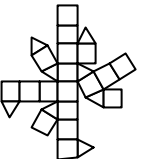
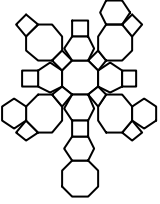
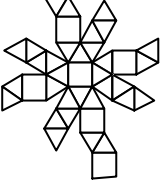
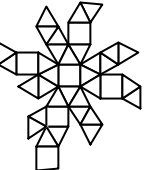
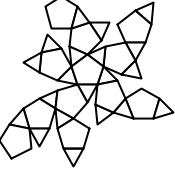
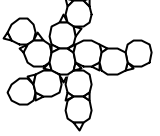

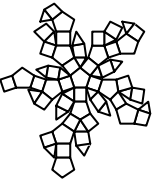
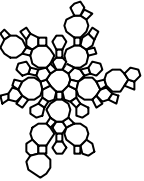

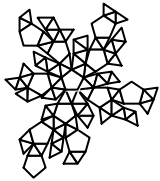
Políedres regulars o platònics (Desenvolupament).

| | | | | |
|---|---|---|---|---|
|  |  |  |  |  |
| Tetràedre | Cub | Octàedre | Dodecàedre | icosàedre |

Políedres arquimedians

| | | | | |
|---|---|---|---|---|
|  |  |  |  |  |
| Tetràedre truncat | Cuboctàedre | Cub truncat | Octàedre truncat | Rombicuboctàedre |
|  |  |  |  |  |
| Gran rombicuboctàedre | Cub simus | Cub simus* | Icosidodecàedre | Dodecàedre truncat |
|  |  |  |  |  |
| Icosàedre truncat | Rombicosidodecàedre | Gran rombicosidodecàedre | Dodecàedre simus | Dodecàedre simus* |

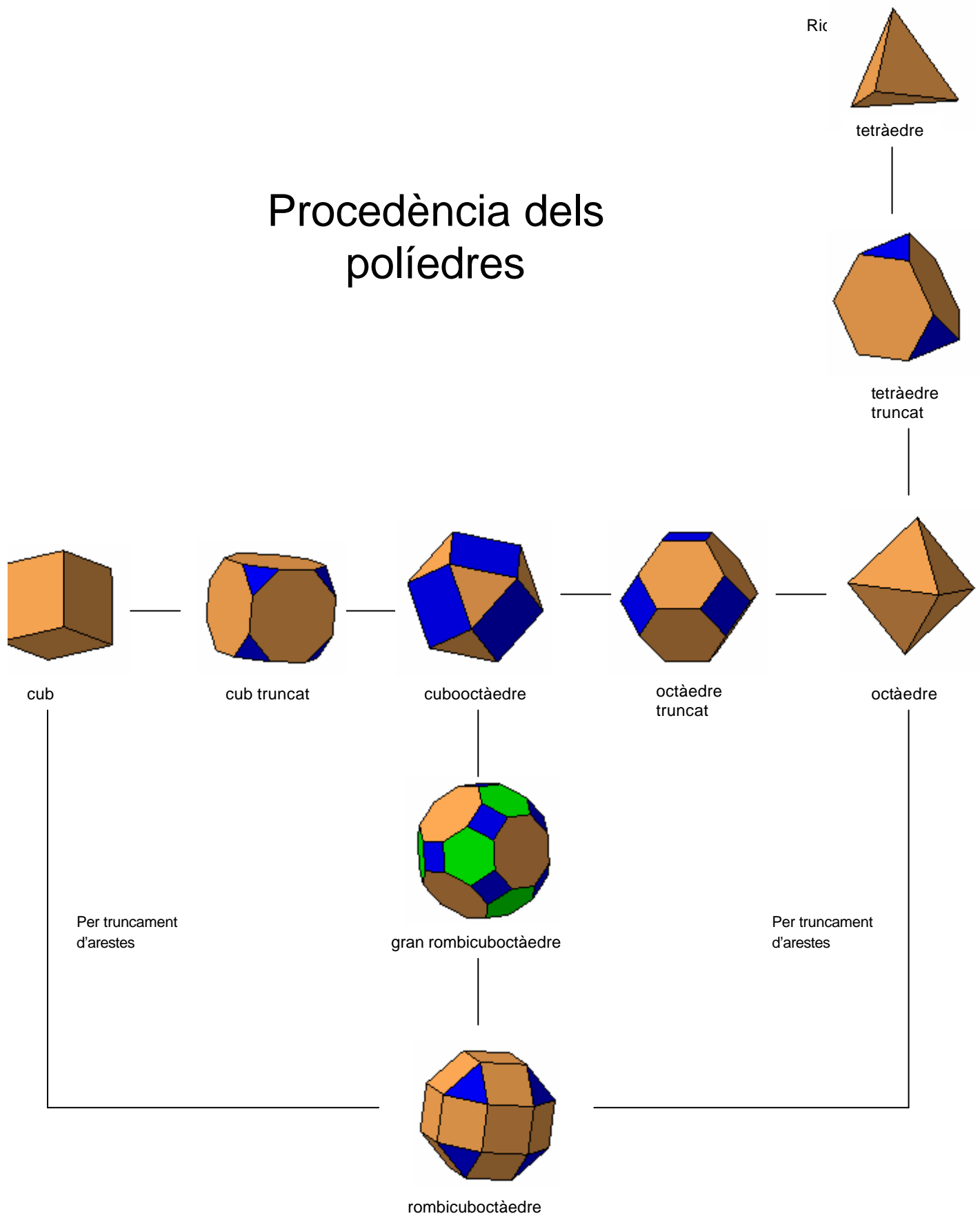
Políedres arquimedians (desenvolupaments).

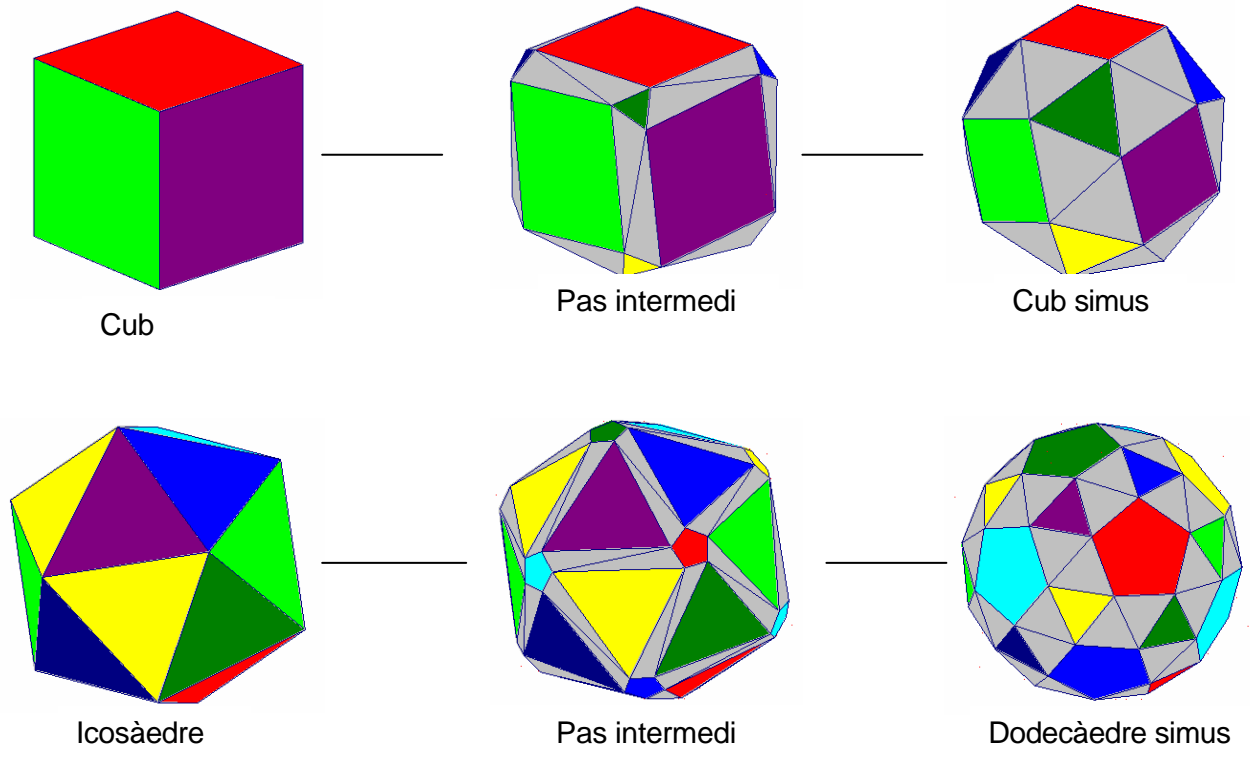
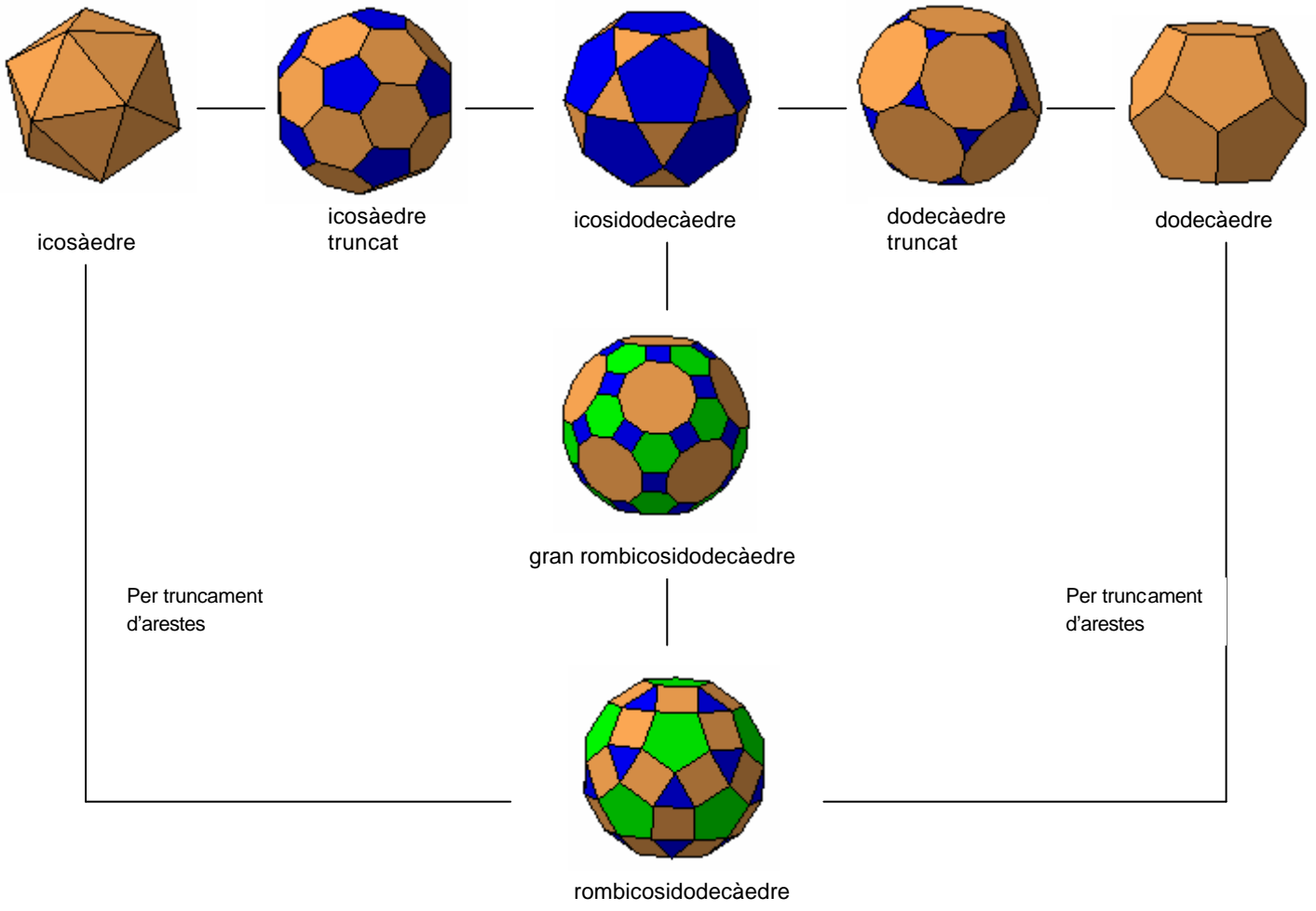
| | | | | |
|--|--|--|--|--|
|  |  |  |  |  |
| Tetràedre truncat | Cuboctàedre | Cub truncat | Octàedre truncat | Rombicuboctàedre |
|  |  |  |  |  |
| Gran rombicuboctàedre | Cub simus | Cub simus* | Icosidodecàedre | Dodecàedre truncat |
|  |  |  |  |  |
| Icosàedre truncat | Rombicosidodecàedre | Gran rombicosidodecàedre | Dodecàedre simus | Dodecàedre simus* |

Procedència dels poliedres arquimedians


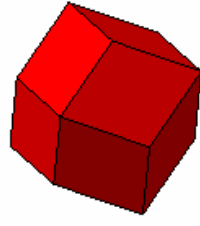
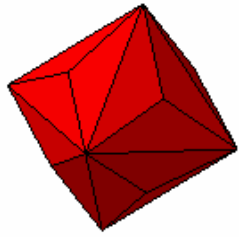
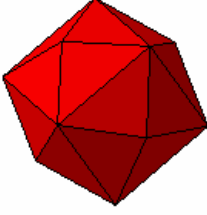
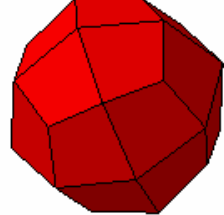
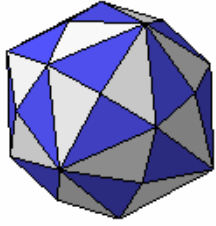
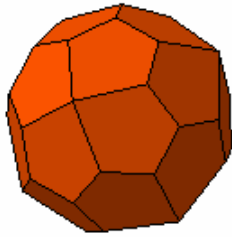

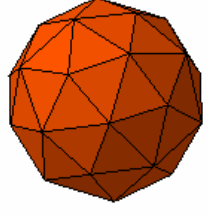
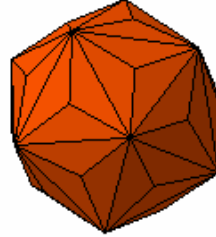
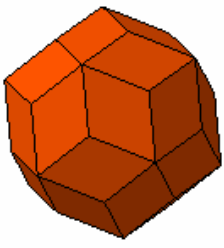
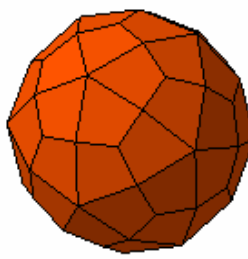
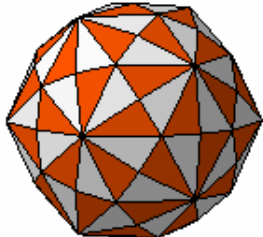
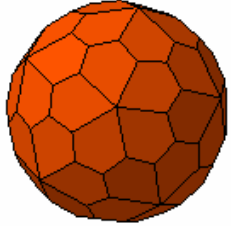
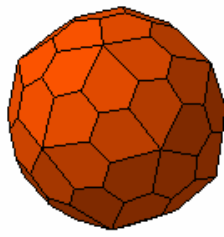
| Poliedres arquimedians | Procedència |
|----------------------------|---|
| Tetràedre truncat | Truncament per $\frac{1}{3}$ de l'aresta d'un tetràedre |
| Cuboctàedre | <ol style="list-style-type: none"> 1. Truncament per $\frac{1}{2}$ de l'aresta d'un cub 2. Truncament per $\frac{1}{2}$ de l'aresta d'un octàedre |
| Cub truncat | Truncament per $\frac{2-\sqrt{2}}{2}$ de l'aresta d'un cub |
| Octàedre truncat | Truncament per $\frac{1}{3}$ de l'aresta d'un octàedre |
| Rombicuboctàedre | Truncament per $\frac{1}{2}$ de l'aresta d'un cuboctàedre |
| Gran rombicuboctàedre | Truncament per $\frac{1}{3}$ de l'aresta d'un cuboctàedre |
| Cub simus o cub aplatat | Truncament i bisellat d'un cub. Com no té plànols de simetria, pot aparèixer amb dues formes simètriques. |
| Icosidodecàedre | <ol style="list-style-type: none"> 1. Truncament per $\frac{1}{2}$ de l'aresta d'un dodecàedre 2. Truncament per $\frac{1}{2}$ de l'aresta d'un icosaèdre |
| Dodecàedre truncat | Truncament per $\frac{5-\sqrt{5}}{10}$ de l'aresta d'un dodecàedre |
| Icosaèdre truncat | Truncament per $\frac{1}{3}$ de l'aresta d'un icosaèdre |
| Rombicosidodecàedre | Truncament per $\frac{1}{2}$ de l'aresta d'un icosidodecàedre |
| Gran rombicosidodecàedre | Truncament per $\frac{1}{3}$ de l'aresta d'un icosidodecàedre |
| Dodecàedre simus o aplatat | Truncament i bisellat d'un icosaèdre. Com no té plànols de simetria, pot aparèixer amb dues formes simètriques. |

Procedència dels políedres

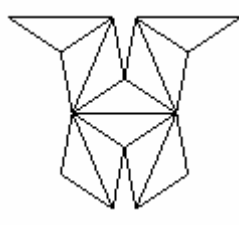
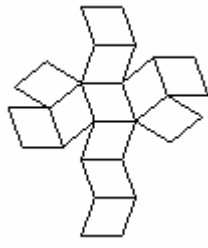
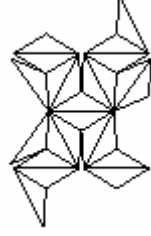
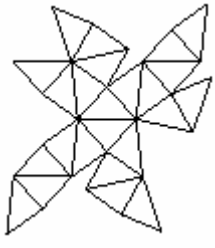
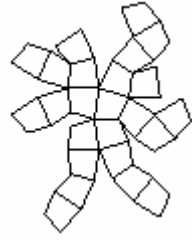
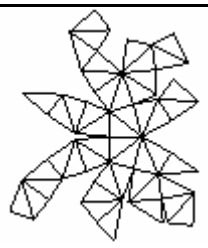
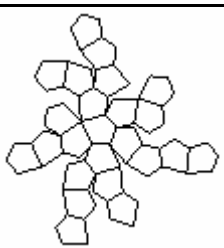
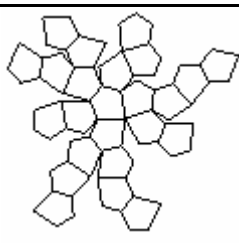
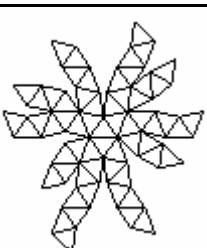
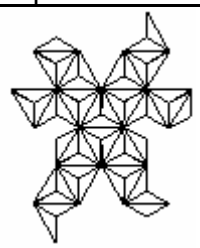


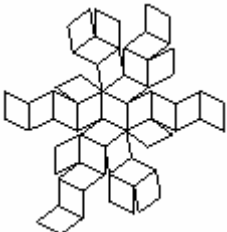
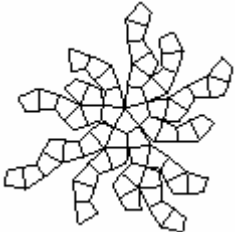

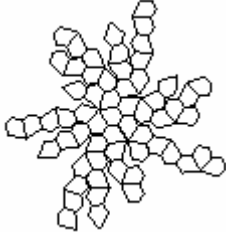



Sòlids de Catalan

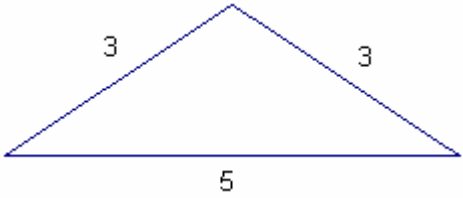
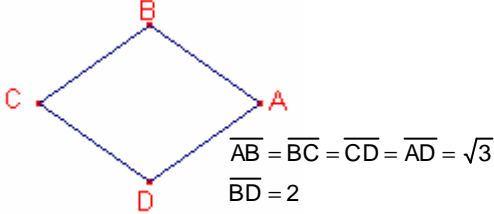
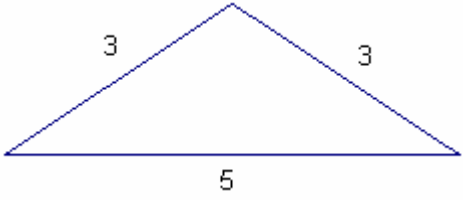
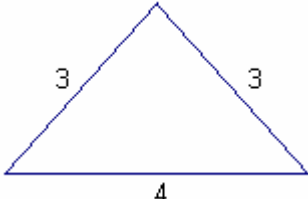
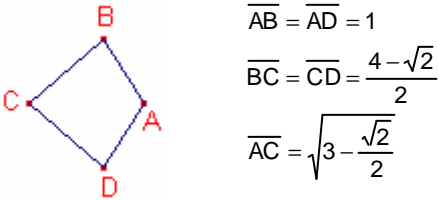
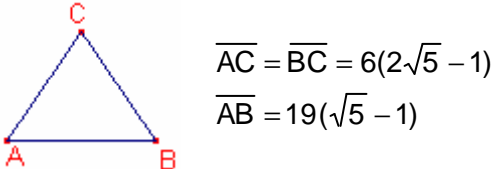
| | | | | |
|--|--|--|--|--|
|  |  |  |  |  |
| Tetraèdre Triakis | Docecàedre ròmbic | Octàedre Triakis | Hexàedre tetrakis | Icositetràedre trapezoidal |
|  |  |  |  |  |
| Dodecàedre pentakis | Icositetràedre pentagonal | Icositetràedre pentagonal* | Dodecàedre pentakis | Icosàedre triakis |
|  |  |  |  |  |
| Triacontàedre ròmbic | Hexacontàedre trapezoidal | Icosàedre hexàkis | Hexacontàedre pentagonal | Hexacontàedre pentagonal* |

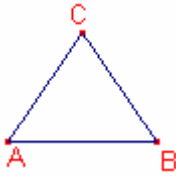
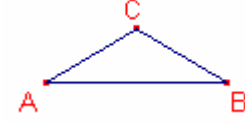
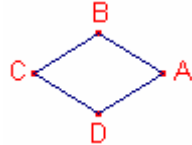
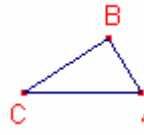
Políedres de Catalan (desenvolupaments)

| | | | | |
|---|---|---|---|---|
|  |  |  |  |  |
| Tetraèdre Triakis | Docecàedre ròmbic | Octàedre Triakis | Hexàedre tetrakis | Icositetràedre trapezoidal |
|  |  |  |  |  |
| Dodecàedre pentakis | Icositetràedre pentagonal | Icositetràedre pentagonal* | Dodecàedre pentakis | Icosàedre triakis |

| | | | | |
|---|---|---|---|---|
|  |  |  |  |  |
| Triacontàedre ròmbic | Hexacontàedre trapezoïdal | Icosàedre hexakis | Hexacontàedre pentagonal | Hexacontàedre pentagonal* |

Cares, vèrtexs i arestes dels sòlids de Catalan

| | Cares | vèrtexs | Arestes | Polígon de la cara |
|----------------------------|-------|---------|---------|--|
| Tetraèdre triakis | 12 | 8 | 18 |  |
| Dodecàedre ròmbic | 12 | 14 | 24 |  |
| Octàedre triakis | 24 | 14 | 36 |  |
| Hexàedre tetrakis | 24 | 14 | 36 |  |
| Icositetraèdre trapezoidal | 24 | 26 | 48 |  |
| Dodecàedre pentakis | 48 | 26 | 72 |  |

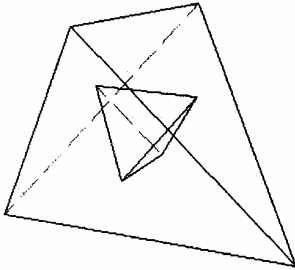
| | | | | |
|---------------------------|-----|----|-----|--|
| Icositetràedre pentagonal | 24 | 38 | 60 | |
| Dodecàedre pentakis | 60 | 32 | 90 |  $\overline{AC} = \overline{BC} = 6(2\sqrt{5} - 1)$ $\overline{AB} = 19(\sqrt{5} - 1)$ |
| Icosàedre triakis | 60 | 32 | 90 |  $\overline{AC} = \overline{BC} = 5(7 + \sqrt{5})$ $\overline{AB} = 11(5 + \sqrt{5})$ |
| Triacontàedre ròmbic | 30 | 32 | 60 |  $\overline{AB} = \overline{AD} = \overline{BC} = \overline{CD} = 1$ $\overline{AC} = \frac{500^{1/4}(1+\sqrt{5})\sqrt{-1+\sqrt{5}}}{10}$ |
| Hexacontàedre trapezoïdal | 60 | 62 | 120 | |
| Icosàedre hexakis | 120 | 62 | 180 |  $\overline{AB} = \sqrt{1275 - 465\sqrt{5}}$ $\overline{BC} = 3\sqrt{39 + \frac{57}{\sqrt{5}}}$ $\overline{AC} = 11\sqrt{12 - \frac{12}{\sqrt{5}}}$ |
| Hexacontàedre pentagonal | 60 | 92 | 150 | |

Políedres duals

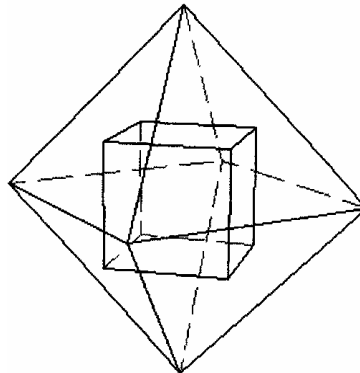
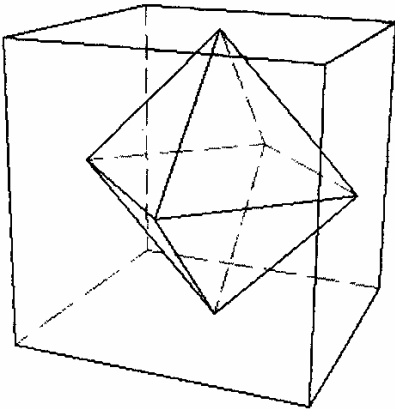
Si en un políedre unim entre si els centres de les cares, obtenim un altre políedre el nombre de cares del qual coincideix amb el nombre de vèrtexs del primer i viceversa. A aquests políedres s'anomenen duals.

Políedres duals dels sòlids platònics.

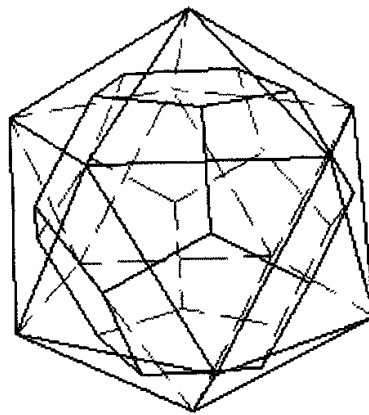
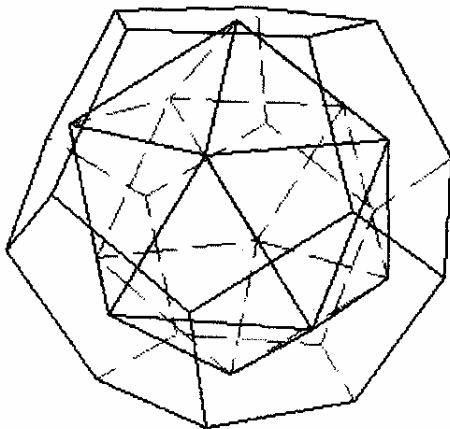
Tetràedre-Tetràedre



Cub-Octàedre

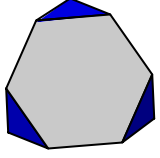

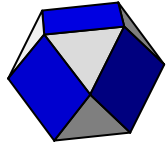
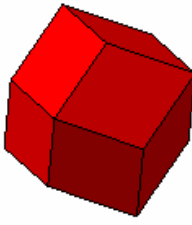
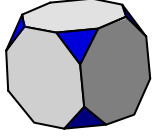
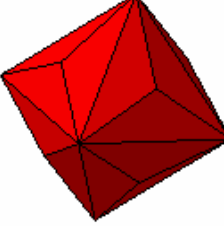
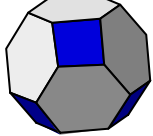
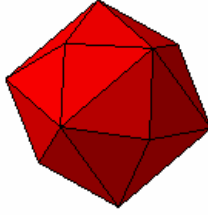
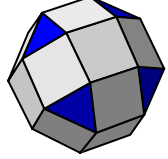
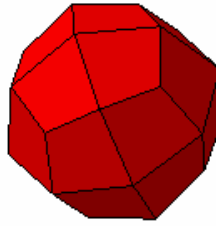
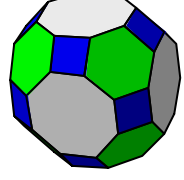
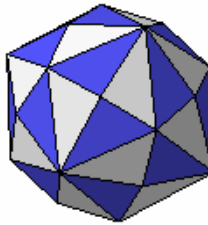


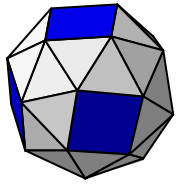
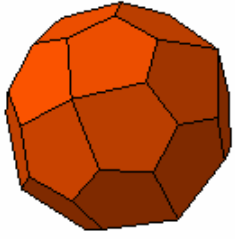
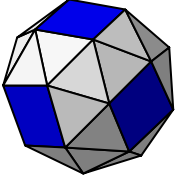
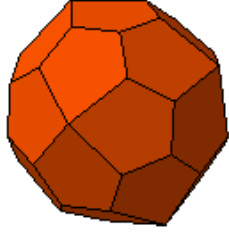
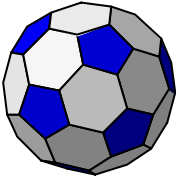
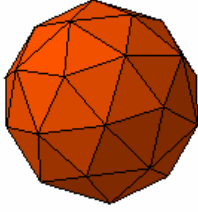
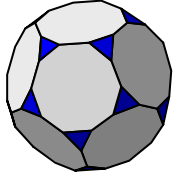

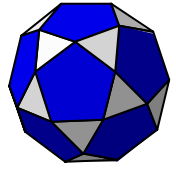
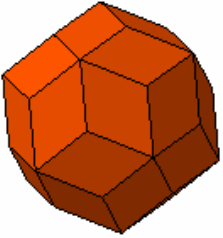
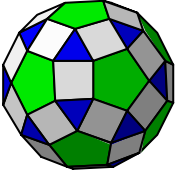
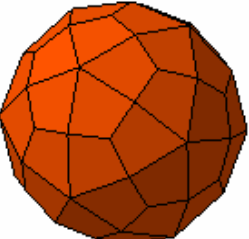
Dodecàedre-Icosàedre.

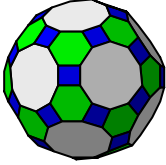
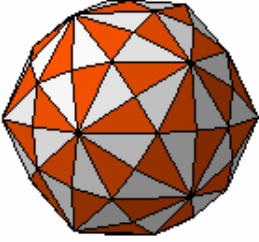
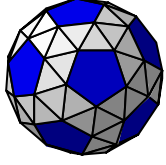

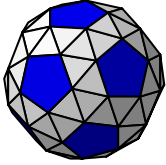
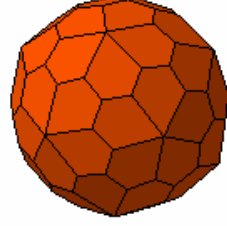


Els políedres duals dels políedres arquimedians són els sòlids o políedres de Catalan.

Políedres arquimedians i els duals de Catalan:

| | Políedres Arquimedians | Políedres de Catalan | |
|-----------------------|---|--|---------------------------|
| Tetraèdre truncat |  |  | Tetraèdre Triakis |
| Cubooctaèdre |  |  | Dodecàedre ròmbic |
| Cub truncat |  |  | Octàedre Triakis |
| Octàedre truncat |  |  | Hexàedre tetrakis |
| Rombicuboctàedre |  |  | Icositrahèdre trapezoidal |
| Gran rombicuboctàedre |  |  | Dodecàedre pentakis |

| | | | |
|---------------------|---|--|-------------------------------|
| Cub Simus |  |  | Icositetràedre pentagonal |
| Cub Simus* |  |  | Icositetràedre pentagonal* |
| Icosàedre truncat |  |  | Dodecaèdre pentakis |
| Dodecaèdre truncat |  |  | Icosàedre triakis |
| Icosidodecaèdre |  |  | Triacontàedre ròmbic |
| Rombicosidodecaèdre |  |  | Hexacontàedre trapezoïdal |

| | | | |
|-----------------------------|---|--|------------------------------|
| Gran rombicosidodecaedre |  |  | Icosàedre hexakis |
| Dodecaèdre simus |  |  | Hexacontàedre pentagonal |
| Dodecaèdre simus* |  |  | Hexacontàedre pentagonal* |

Prismes i antiprismes

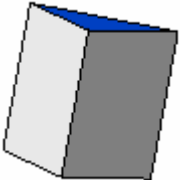
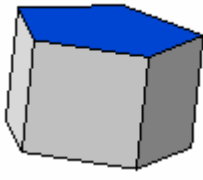
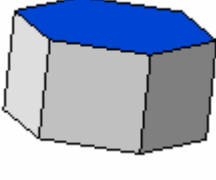
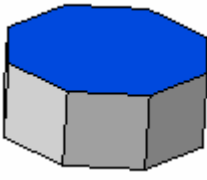
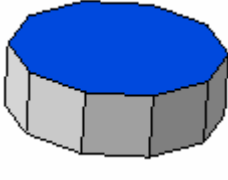
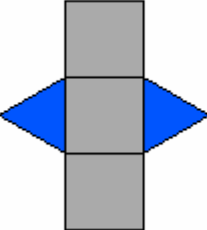
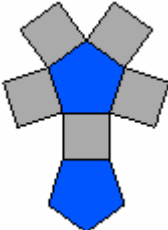
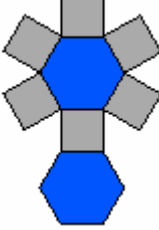
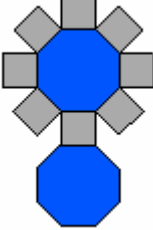
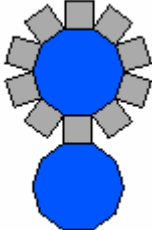
Els prismes són poliedres formats per dues cares poligonals iguals (anomenades bases), paral·leles i disposades en la mateixa orientació (costats homòlegs paral·lels), de forma que al unir els vèrtexs homòlegs d'ambdues cares resulten rectangles o paral·lelograms.

En aquest estudi només considerarem les que formen cares laterals quadrats i bases polígons regulars.

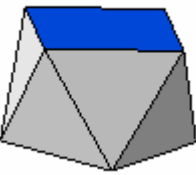
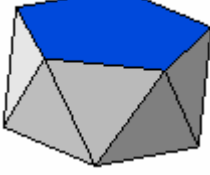
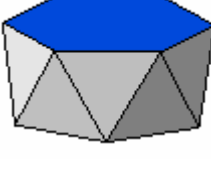
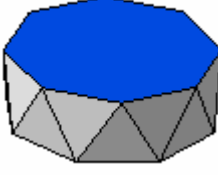
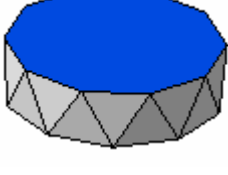
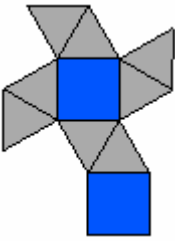
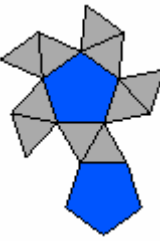
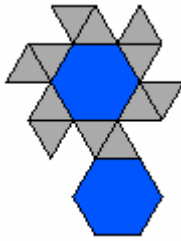
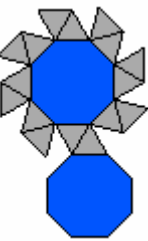
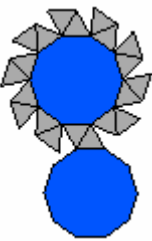
Els antiprismes són poliedres formats per dues cares poligonals iguals i disposades lleugerament girades una respecte de l'altra (costats homòlegs no paral·lels), unint cada vèrtex amb l'altre no homòleg més pròxim s'obtenen cares laterals triangulars iguals alternades en orientacions.

En aquest estudi només considerarem les cares laterals formades per triangles equilàters i bases polígons regulars.

Exemples de prismes i els seus desenvolupaments:

| | | | | |
|---|---|---|--|---|
|  |  |  |  |  |
|  |  |  |  |  |
| Prisma triangular | Prisma pentagonal | Prisma hexagonal | Prisma octogonal | Prisma decagonal |

Exemples d'antiprismes i els seus desenvolupaments:

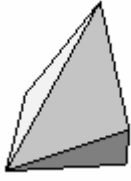
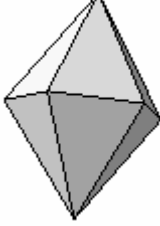
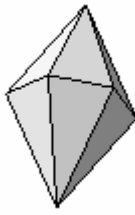


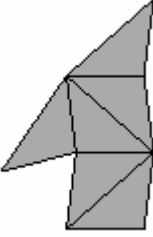
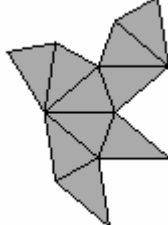
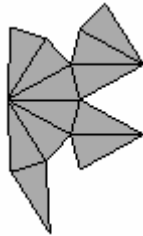
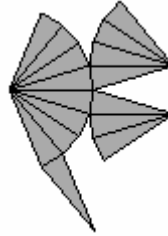
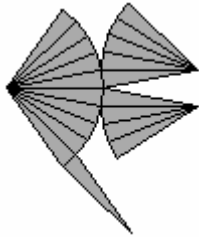
| | | | | |
|---|---|---|--|---|
|  |  |  |  |  |
|  |  |  |  |  |
| Antiprisma quadrangular | Prisma pentagonal | Antiprisma hexagonal | Antiprisma octogonal | Antiprisma decagonal |

Dipiràmides i deltàedres.

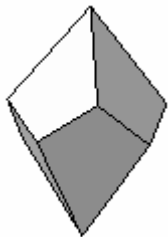




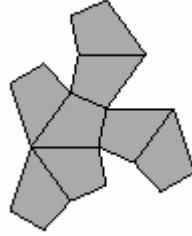
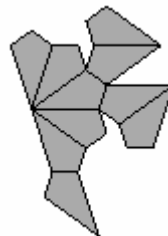
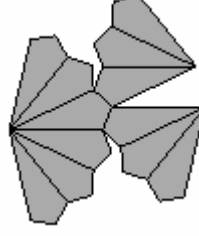
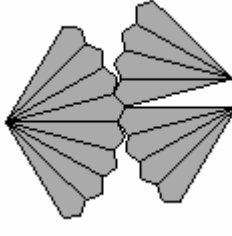
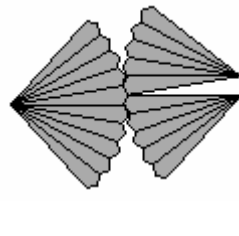
Les dipiràmides són els políedres duals dels prismes.

Els deltàedres són els políedres duals dels antiprismes.

Exemples de dipiràmides (duals dels prismes) i els seus desenvolupaments:

| | | | | |
|---|---|---|--|---|
|  |  |  |  |  |
|  |  |  |  |  |
| Dipiràmide triangular | Dipiràmide pentagonal | Dipiràmide hexagonal | Dipiràmide octogonal | Dipiràmide decagonal |

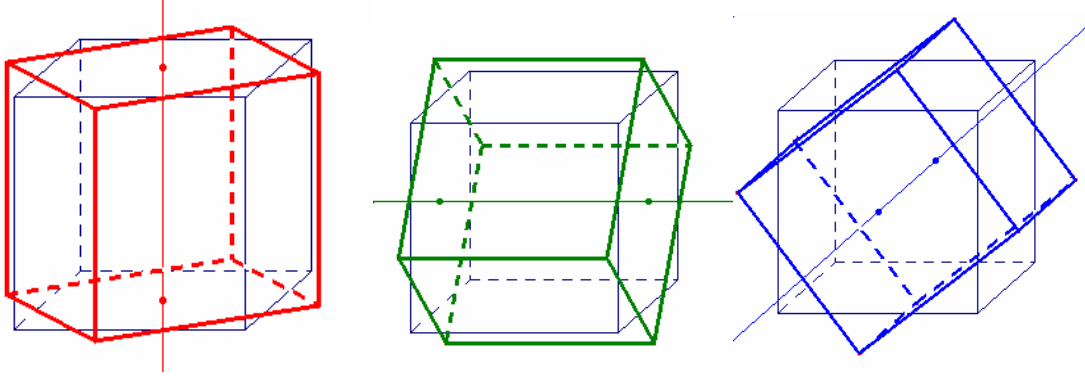
Exemples de deltàedres (duals dels antiprismes) i els seus desenvolupaments:

| | | | | |
|---|---|---|--|---|
|  |  |  |  |  |
|  |  |  |  |  |
| Deltàedres quadrangular | Deltàedres pentagonal | Deltàedres hexagonal | Deltàedres octogonal | Deltàedres decagonal |

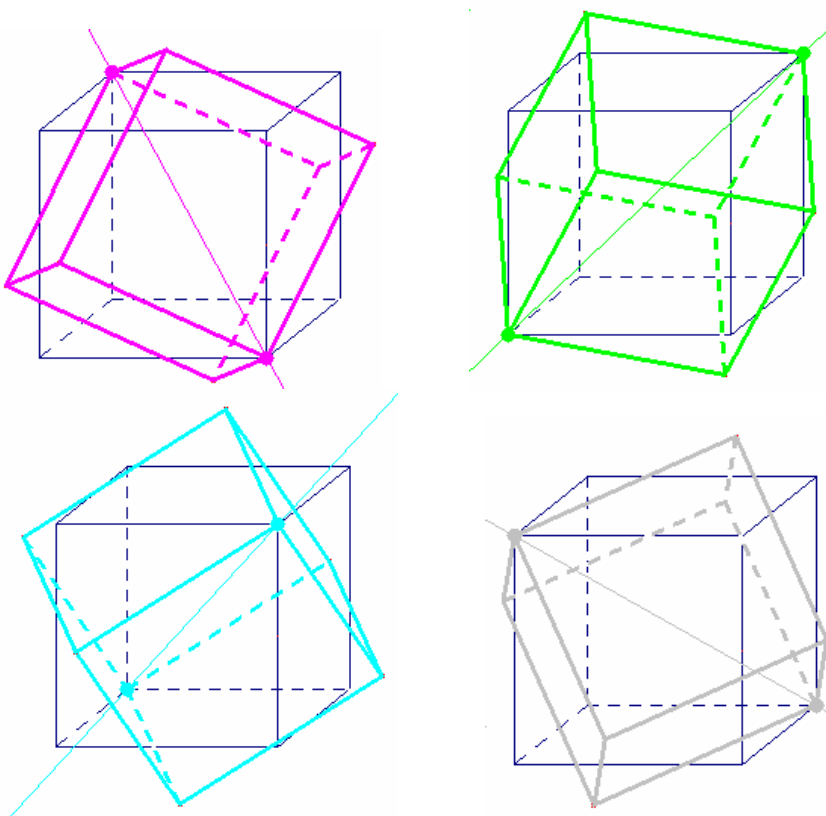
Simetries del cub

Eix de simetria o de rotació és la recta que travessa el políedre i que, si el políedre gira al voltant d'ella, torna a coincidir el políedre abans de donar una volta completa. L'ordre és el nombre de vegades que coincideix el políedre fins a donar una volta completa.

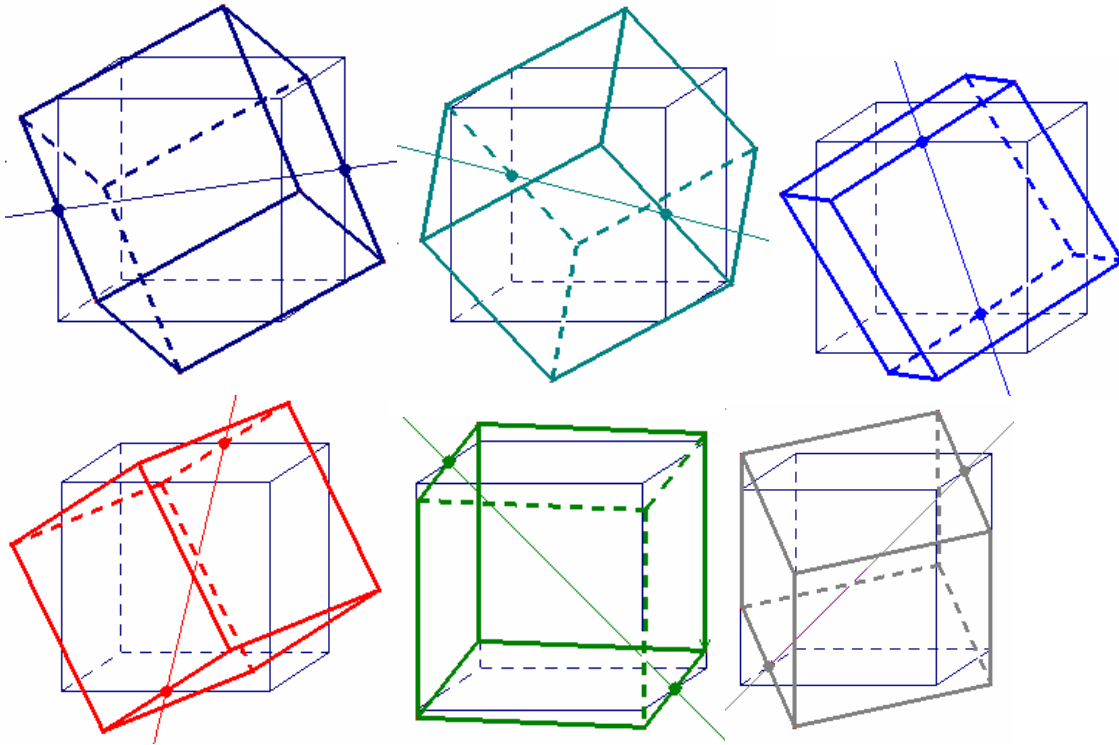
3 eixos de simetria d'ordre 4



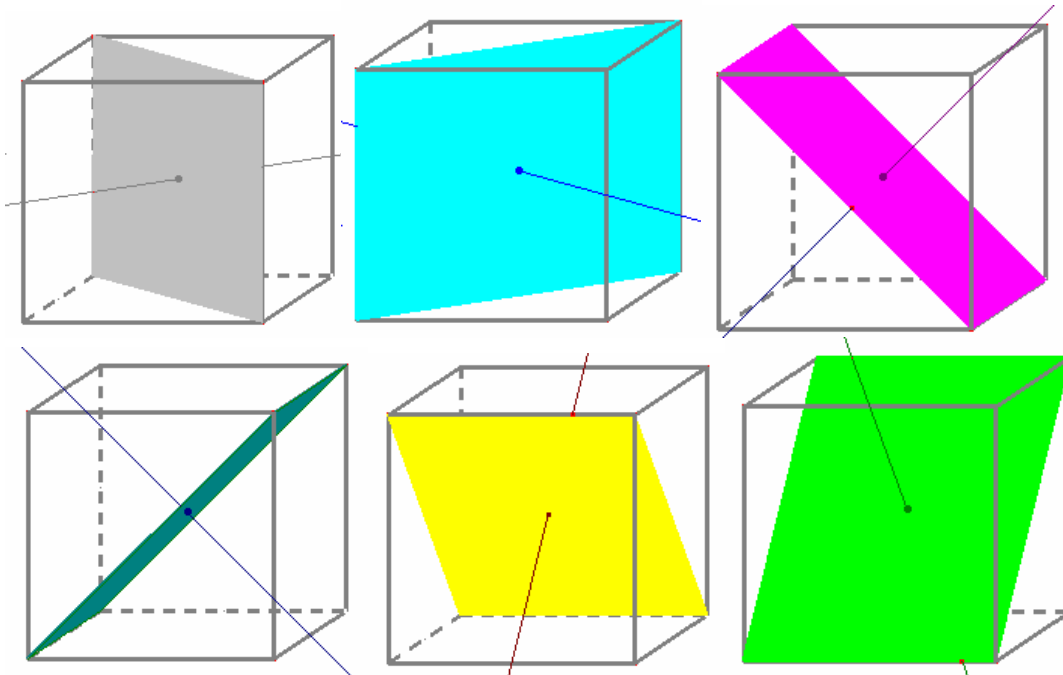
4 eixos de simetria d'ordre 3



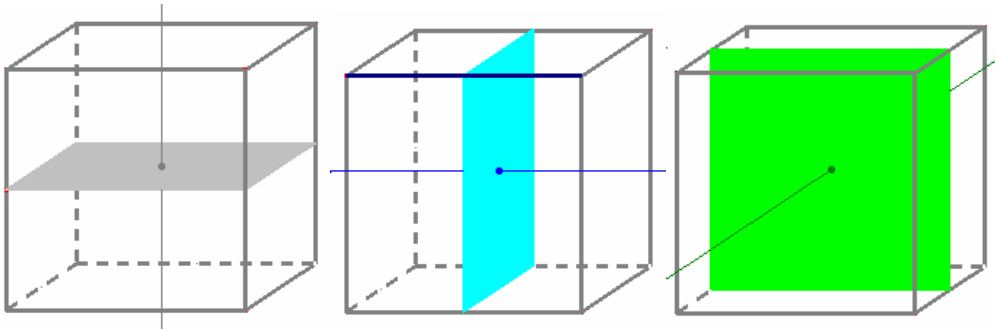
6 eixos de simetria d'ordre 2



6 plànols de simetria perpendiculars als eixos d'ordre 2 que passen pel centre del políedre.

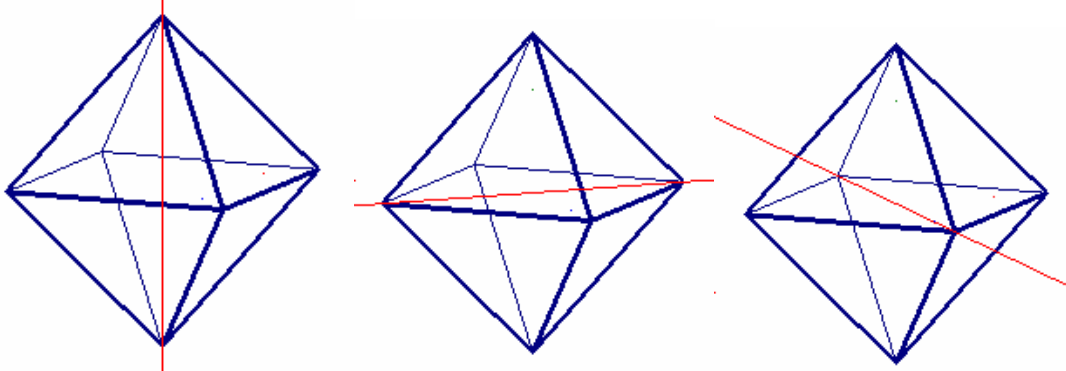


3 plànols de simetria perpendiculars als eixos d'ordre 4 que passen pel centre del políedre.

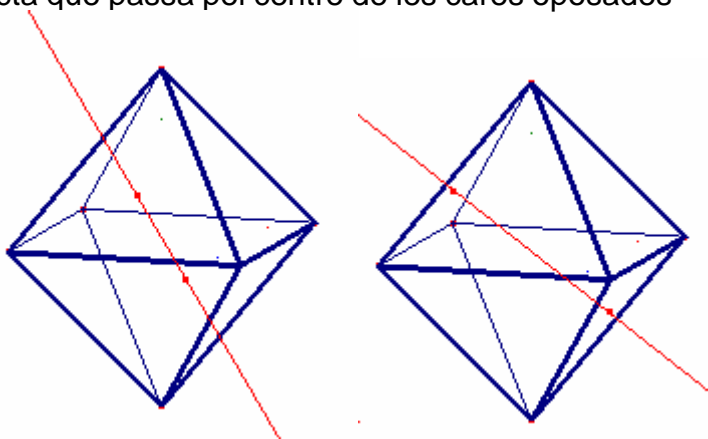


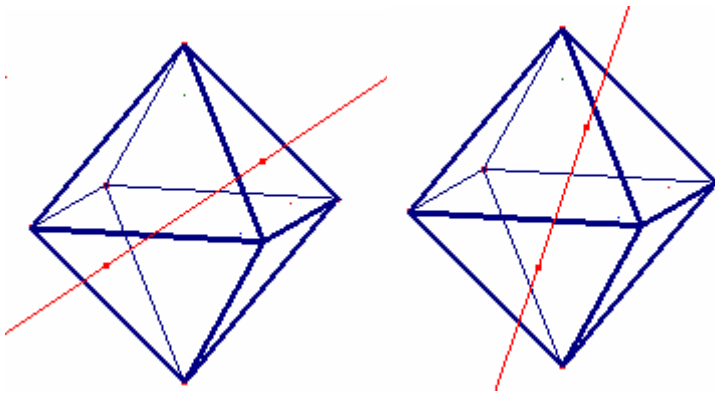
Simetries de l'octàedre

3 eixos de simetria d'ordre 4
recta que passa pels vèrtexs oposats

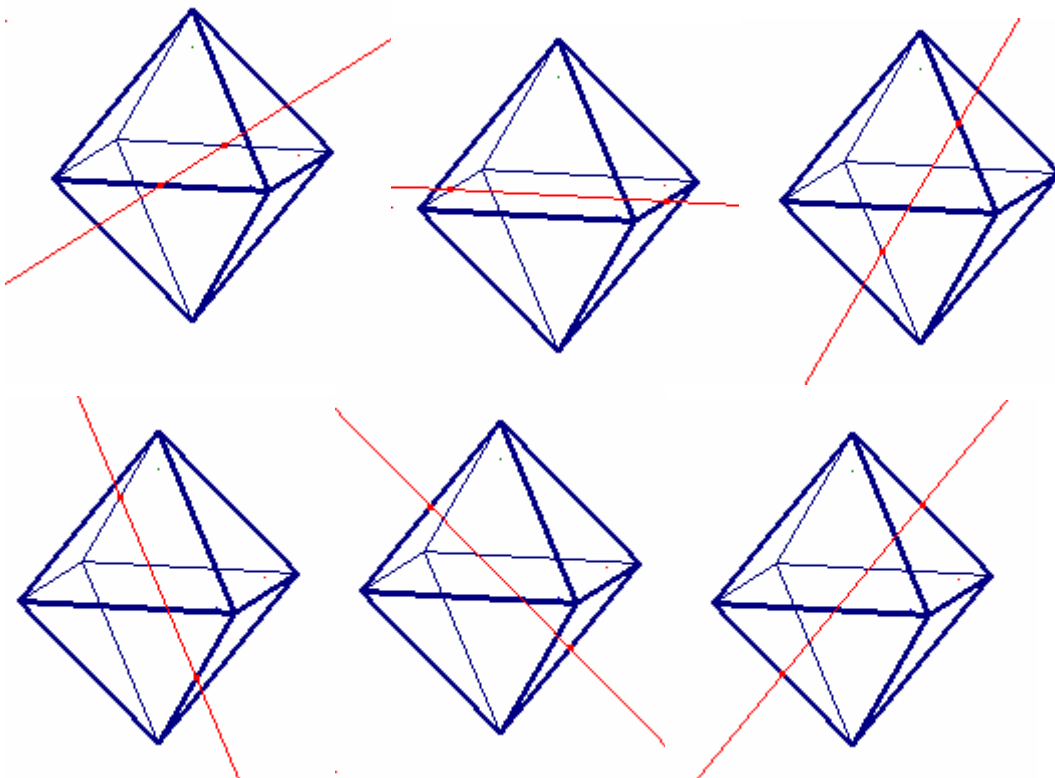


4 eixos de simetria d'ordre 3
Recta que passa pel centre de les cares oposades

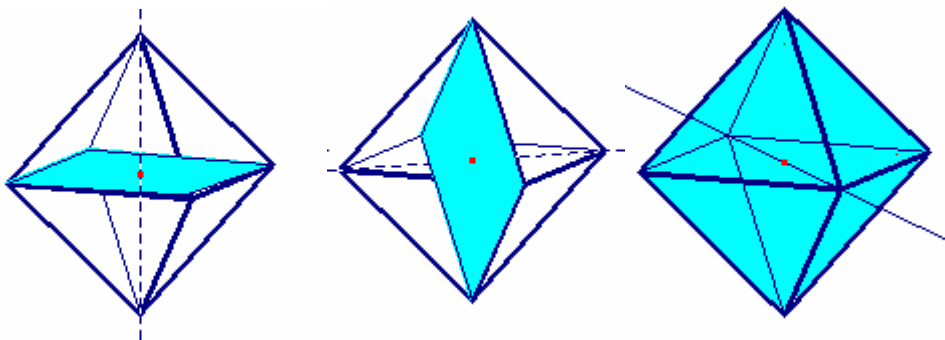




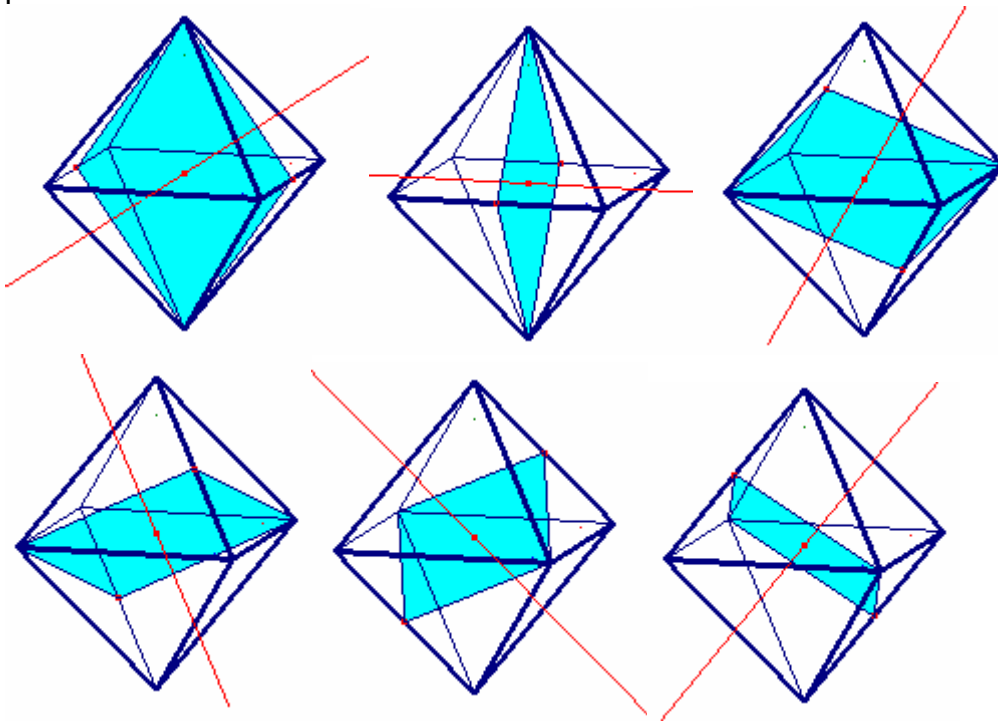
6 eixos de simetria d'ordre 2
recta que passa pel centre de les arestes oposades



3 plànols de simetria perpendiculars als eixos d'ordre 4 que passen pel centre del políedre



6 plànols de simetria perpendiculars als eixos d'ordre 2 que passen pel centre del políedre.



Angles dièdrics.

Angle dièdric és el que formen dues cares que es tallen en una aresta d'un políedre.

Angles dièdrics dels políedres regulars.

| Políedre regular | Angle dièdric |
|------------------|---|
| Tetraèdre | $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$ |
| Cub | 90° |
| Octàedre | $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$ |
| Dodecàedre | $\arccos\left(-\frac{1}{5}\sqrt{5}\right) \approx 116^\circ 33' 54.1''$ |
| Icosàedre | $\arccos\left(-\frac{1}{3}\sqrt{5}\right) \approx 138^\circ 11' 22.8''$ |

Empaquetaments

Anomenem empaquetaments de políedres a l'agrupació de políedres que s'uneixen formant combinacions amb les quals no queden espais lliures entre ells.

S'ha de complir:

- Les cares dels políedres en connexió han de ser del mateix tipus i amb la mateixa longitud del costat.
- La suma dels angles dièdrics que convergeixen en la mateixa cara ha de ser 360° .

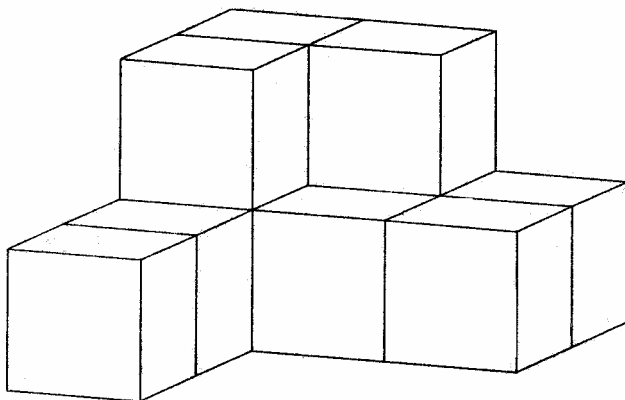
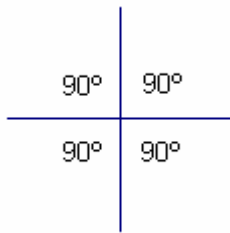
Empaquetament 1

Políedre que el forma: Cub.

Angle dièdric entre les cares:

Quadrat-Quadrat 90° .

Convergeixen 4 cubs per aresta, essent la suma dels quatre angles 360° .



Empaquetament 2

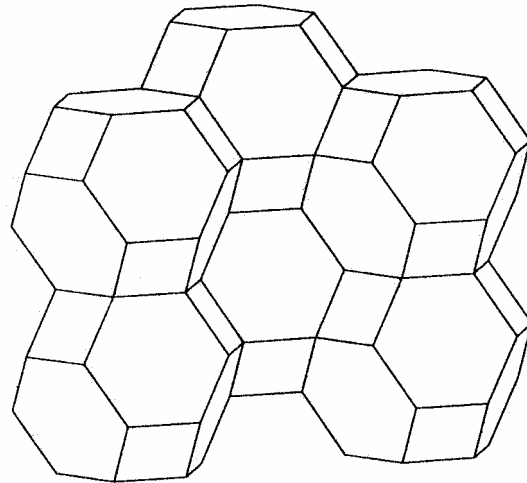
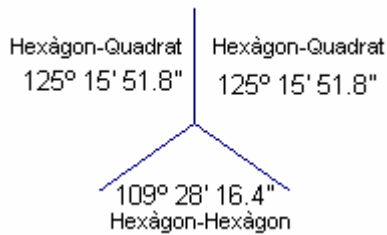
Poliedre que el forma: octàedre truncat.

Angles dièdrics entre les cares:

Hexàgon-Hèxagon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.4''$

Hèxagon-Quadrat $180^\circ - \arccos\left(\frac{\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Convergeixen 3 octàedres truncats per aresta, essent la suma dels quatre angles 360° .



Empaquetament 3

Poliedres que el formen: tetràedre, octàedre.

Angles dièdrics entre les cares:

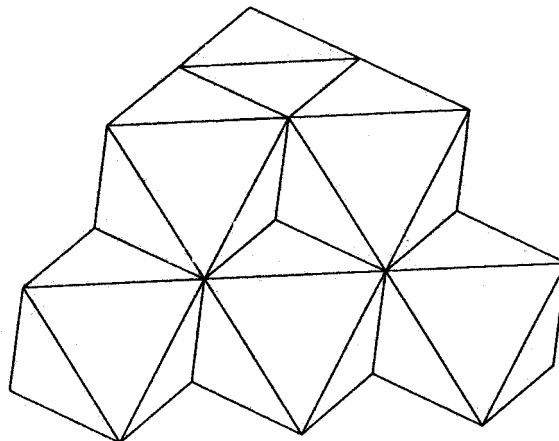
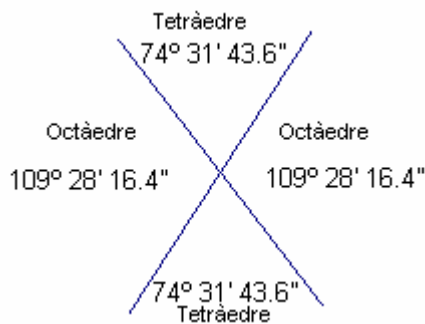
Tetràedre:

Triangle-triangle $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$

Octàedre:

Triangle-Triangle $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

Convergeixen 2 poliedres de forma alternada (2 tetràedres i 2 octàedres) convergent en una mateixa aresta, essent la suma dels quatre angles 360° .



Empaquetament 4

Poliedres que el formen: tetràedre, tetràedre truncat.

Angles dièdrics entre les cares:

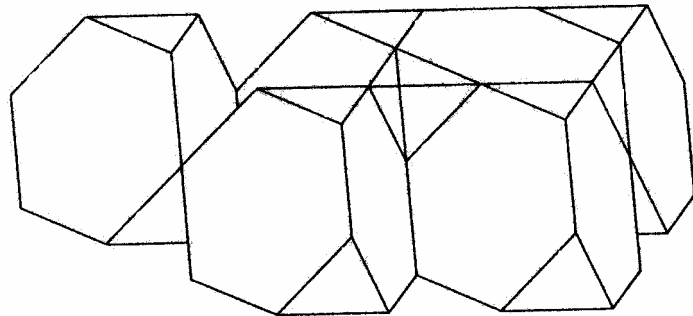
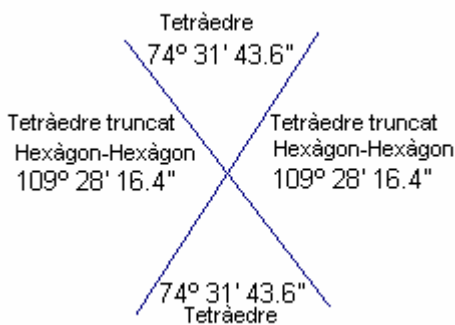
Tetràedre:

Triangle-triangle $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$

tetràedre truncat:

Hexàgon-Hexàgon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

Convergeixen 2 poliedres de forma alternada (2 tetràedres i 2 tetràedres truncats) convergent en una mateixa aresta, essent la suma dels quatre angles 360° .



Empaquetament 5

Poliedres que el formen: octàedre, Cubooctàedre.

Angles dièdrics entre les cares:

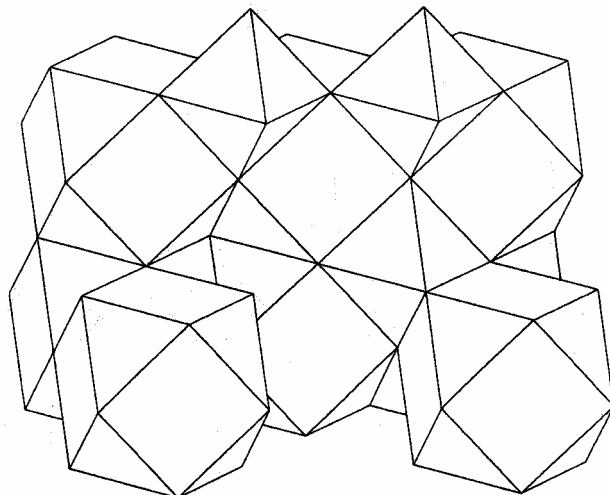
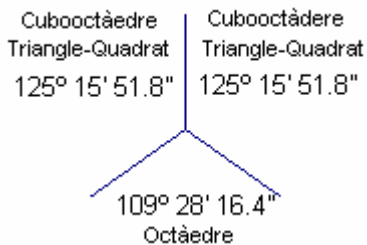
octàedre:

Triangle-Triangle $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

cubooctàedre:

Triangle-Quadrat $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Convergeixen 1 octàedre i 2 cubooctàedres en una mateixa aresta, essent la suma dels quatre angles 360° .



Empaquetament 6

Políedres que el formen: octàedre, cub truncat.

Angles dièdrics entre les cares:

Octàedre:

Triangle-Triangle $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

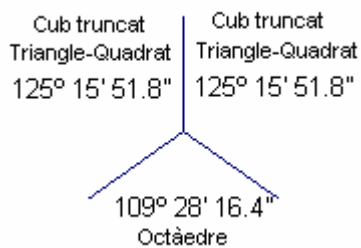
Cub truncat:

Octògon-Octògon 90°

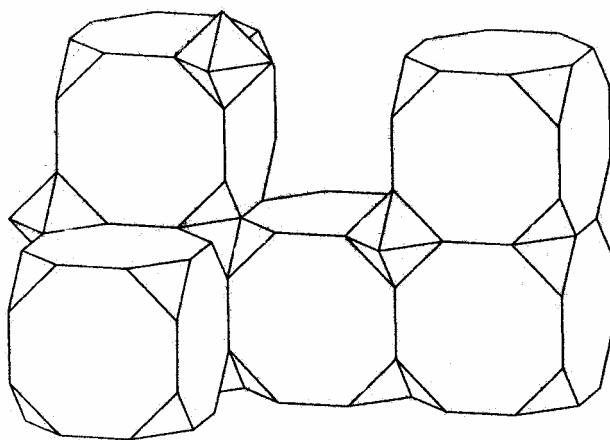
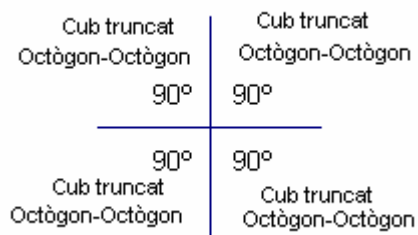
Triangle-Octògon $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Es donen 2 casos de connexió:

- convergeixen un octàedre i dos cubs truncats.



- convergeixen quatre cubs truncats.



Empaquetament 7

Poliedres que el formen: tetràedre, cub, rombicuboctàedre.

Angles dièdrics entre les cares:

Tetràedre:

Triangle-triangle $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$

Cub:

90°

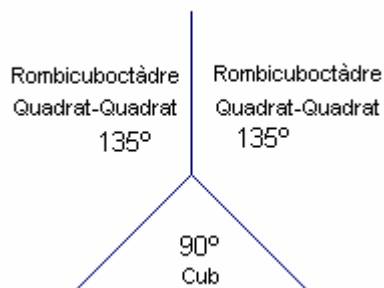
Rombicuboctàedre:

Triangle-Quadrat $\arctg\left(\frac{-\sqrt{2}}{2}\right) \approx 144^\circ 44' 8.2''$

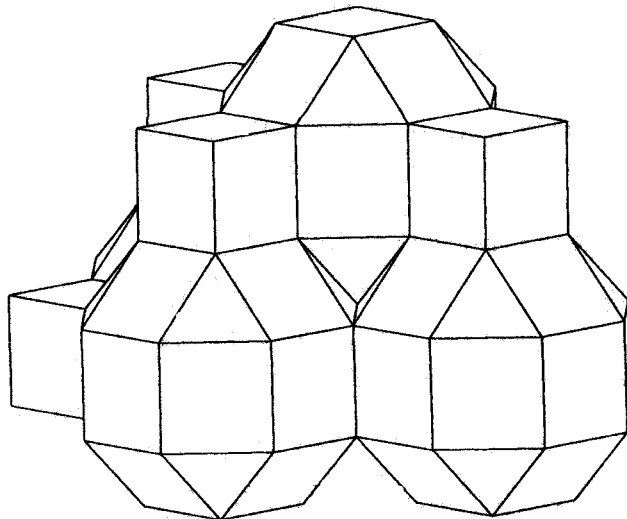
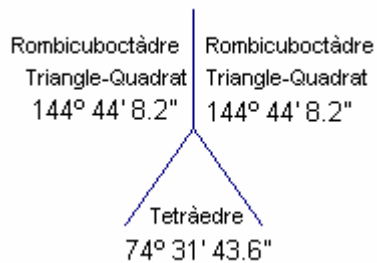
Quadrat-Quadrat 135°

Es donen 2 casos de connexió:

1. Convergeixen un cub i dos rombicuboctàedres per aresta.



2. Convergeixen un tetràedre i dos rombicuboctàedres per aresta.



Empaquetament 8

Poliedres que el formen: cub, cubooctàedre, rombicuboctàedre.

Angles dièdrics entre les cares:

Cub:

90°

Cubooctàedre:

Triangle-Quadrat $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

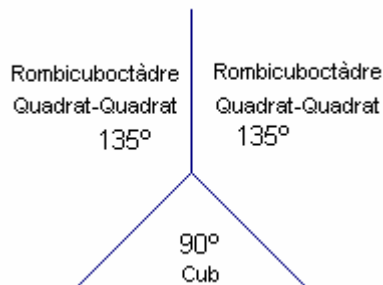
Rombicuboctàedre:

Triangle-Quadrat $\arctg\left(\frac{-\sqrt{2}}{2}\right) \approx 144^\circ 44' 8.2''$

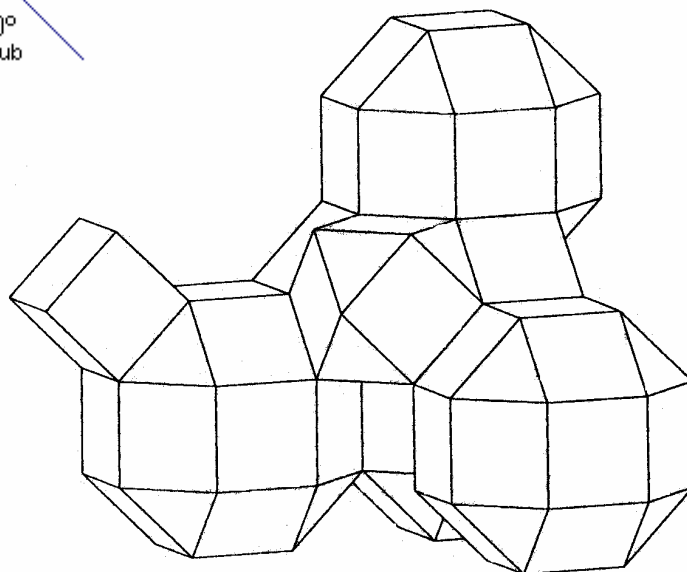
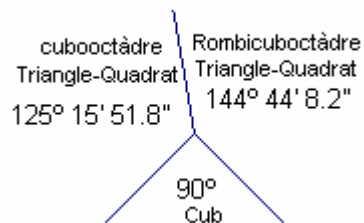
Quadrat-Quadrat 135°

Es donen 2 casos de connexió:

1. Convergeixen un cub i dos rombicuboctàedres per aresta.



2. Convergeixen un tetràedre un cubooctàedre, i un rombicuboctàedre per aresta.



Empaquetament 9

Poliedres que el formen: cub, octàedre truncat, gran rombicuboctàedre.

Angles dièdrics entre les cares:

Cub:

90°

Octàedre truncat:

Hexàgon-Hèxagon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.4''$

Hèxagon-Quadrat $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Gran rombicuboctàedre:

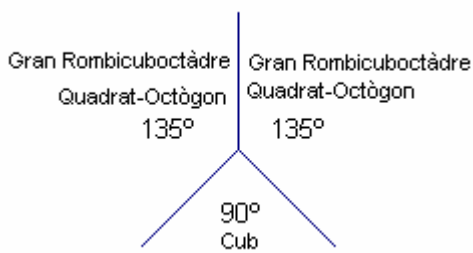
Hexàgon-Octògon $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Hexàgon-Quadrat $\arctg\left(\frac{-\sqrt{2}}{2}\right) \approx 144^\circ 44' 8.2''$

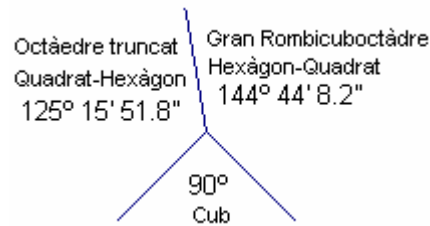
Quadrat-Octògon 135°

Es donen 3 casos de connexió:

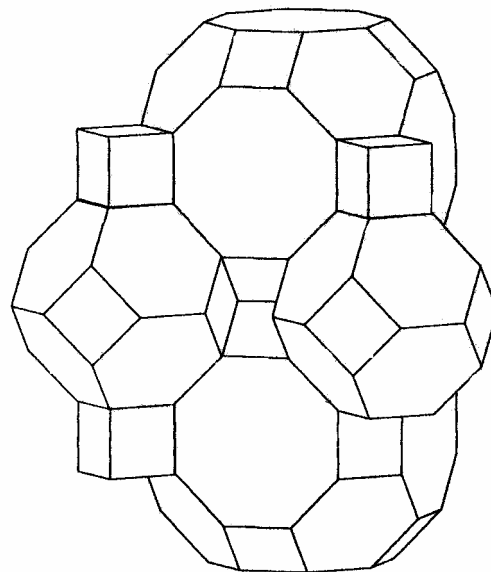
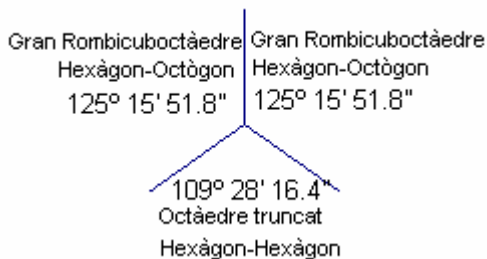
1. Convergeixen un cub i dos grans rombicuboctàedres per aresta.



2. Convergeixen un cub un octàedre truncat i un gran rombicuboctàedre per aresta.



3. Convergeixen un octàedre truncat i dos grans rombicuboctàedres per aresta.



Empaquetament 10

Políedres que el formen: tetràedre truncat, cub truncat, gran rombicuboctàedre.

Angles dièdrics entre les cares:

Tetràedre truncat:

Triangle-Hexàgon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

Hexàgon-Hexàgon $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$

Cub truncat:

Octògon-Octògon 90°

Triangle-Octògon $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Gran rombicuboctàedre:

Hexàgon-Octògon $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Hexàgon-Quadrat $\arctg\left(\frac{-\sqrt{2}}{2}\right) \approx 144^\circ 44' 8.2''$

Quadrat-Octògon 135°

Es donen 3 casos de connexió:

| | |
|---|---|
| <p>1. Convergeixen un tetràedre truncat i dos grans rombicuboctàedres per aresta.</p> | <p>2. Convergeixen un tetràedre truncat un cub truncat i un gran rombicuboctàedre per aresta.</p> |
| <p>3. Convergeixen un cub truncat i dos grans rombicuboctàedres per aresta.</p> | |

Empaquetament 11

Poliedres que el formen: tetràedre truncat, octàedre truncat, cubooctàedre.

Angles dièdrics entre les cares:

Tetràedre truncat:

Triangle-Hexàgon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.39''$

Hexàgon-Hexàgon $\arccos\left(\frac{1}{3}\right) \approx 70^\circ 31' 43.61''$

Octàedre truncat:

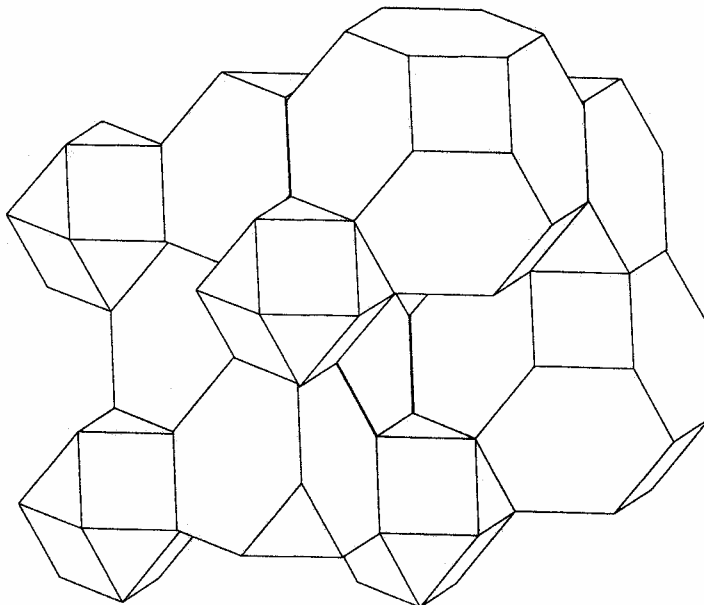
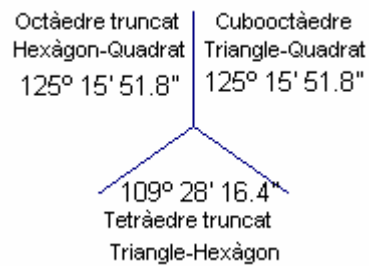
Hexàgon-Hèxagon $\arccos\left(\frac{-1}{3}\right) \approx 109^\circ 28' 16.4''$

Hèxagon-Quadrat $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Cubooctàedre:

Triangle-Quadrat $\arccos\left(\frac{-\sqrt{3}}{3}\right) \approx 125^\circ 15' 51.8''$

Convergeixen un tetràedre truncat, un octàedre truncat i un cubooctàedre per aresta.



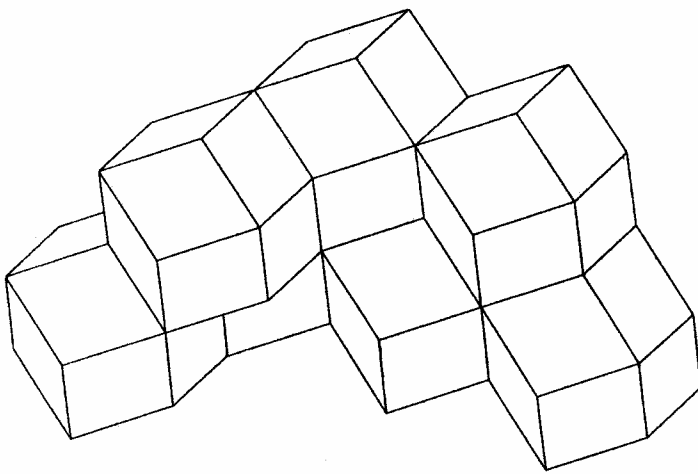
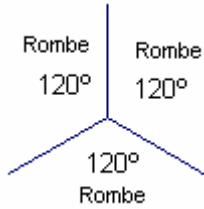
Empaquetament 12

Poliedre que el forma: dodecàedre ròmbic (sòlid de Catalan).

Angles dièdrics entre les cares:

Rombe-Rombe 120°

Convergeixen 3 dodecàedres ròmbics per aresta.



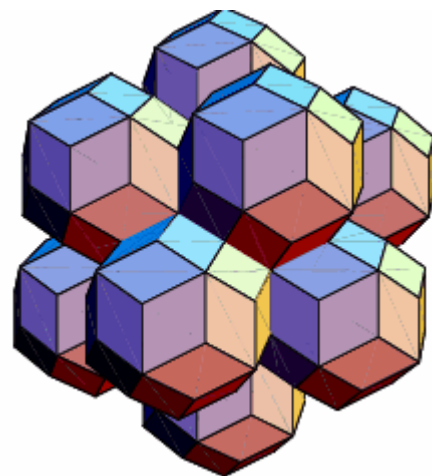
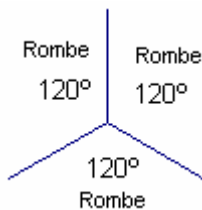
Empaquetament 13

Poliedre que el forma: triacontàedre ròmbic (sòlid de Catalan).

Angles dièdrics entre les cares:

Rombe-Rombe 120°

Convergeixen 3 dodecàedres ròmbics per aresta.



Políedres regulars o platònics:

| Nom | Orde del vèrtex | Cares | Vèrtexs | Arestes | Area | Volum | R radi esfera Circumscrita | r radi esfera tangent arestes | r radi esfera Inscrita | Angle díedre |
|------------|-----------------|-------|---------|---------|---------------------------------|-----------------------------------|---------------------------------------|----------------------------------|---|--------------|
| Tetràedre | 3 | 4 T | 4 | 6 | $A = a^2 \sqrt{3}$ | $V = \frac{a^3 \sqrt{2}}{12}$ | $R = \frac{a\sqrt{6}}{4}$ | $\rho = \frac{a\sqrt{2}}{4}$ | $r = \frac{a\sqrt{6}}{12}$ | 70°31'44" |
| Cub | 3 | 6 Q | 8 | 12 | $A = 6a^2$ | $V = a^3$ | $R = \frac{a\sqrt{3}}{2}$ | $\rho = \frac{a\sqrt{2}}{2}$ | $r = \frac{a}{2}$ | 90° |
| Octàedre | 4 | 8 T | 6 | 12 | $A = 2a^2 \sqrt{3}$ | $V = \frac{a^3 \sqrt{2}}{3}$ | $R = \frac{a\sqrt{2}}{2}$ | $\rho = \frac{a}{2}$ | $r = \frac{a\sqrt{6}}{6}$ | 109°28'16" |
| Dodecàedre | 3 | 12 P | 20 | 30 | $A = 3a^2 \sqrt{25+10\sqrt{5}}$ | $V = \frac{a^3}{4}(15+7\sqrt{5})$ | $R = \frac{a}{4}(\sqrt{15}+\sqrt{3})$ | $\rho = \frac{a(3+\sqrt{5})}{4}$ | $r = \frac{a}{2} \sqrt{\frac{25+11\sqrt{5}}{10}}$ | 116°33'54" |
| Icosàedre | 5 | 20 T | 12 | 30 | $A = 5a^2 \sqrt{3}$ | $V = \frac{5a^3}{12}(3+\sqrt{5})$ | $R = \frac{a}{4}\sqrt{10+2\sqrt{5}}$ | $\rho = \frac{a(1+\sqrt{5})}{4}$ | $r = \frac{a}{12}(3\sqrt{3}+\sqrt{15})$ | 138°11'23" |

Propietat dels radis de les esferes circumscrita, inscrita i tangent a les arestes del políedre.

$$R \cdot r = \rho^2$$

T triangles equilàters.

Q quadrats

P pentàgons regulars

a aresta

Políedres arquimedians

| Nom | Cares | Vèrtexs | Arestes | Àrea | Volum | R radi esfera Circumscrita | r radi esfera tangent arestes | r radi esfera Inscrita |
|--------------------------|-----------------|---------|---------|----------|----------|---------------------------------------|--|---|
| Tetraèdre truncat | 8=4T + 4H | 12 | 18 | 12'1243 | 2'7106 | $R = \frac{a\sqrt{22}}{4}$ | $\rho = \frac{3a\sqrt{2}}{4}$ | $r = \frac{9a\sqrt{22}}{44}$ |
| Cubooctàedre | 14=8T+6Q | 12 | 24 | 9'4641 | 2'3570 | $R = a$ | $\rho = \frac{a\sqrt{3}}{2}$ | $r = \frac{3a}{4}$ |
| Cub truncat | 14=8T + 6 O | 24 | 36 | 32'4346 | 13'5996 | $R = \frac{a\sqrt{7+4\sqrt{2}}}{2}$ | $\rho = \frac{a(2+\sqrt{2})}{2}$ | $r = \frac{a(5+2\sqrt{2})\sqrt{7+4\sqrt{2}}}{17}$ |
| Octàedre truncat | 14=8H + 6Q | 24 | 36 | 26'7846 | 11'3137 | $R = \frac{a\sqrt{10}}{2}$ | $\rho = \frac{3a}{2}$ | $r = \frac{9a\sqrt{10}}{20}$ |
| Rombicuboctàedre | 26=8T + 18Q | 24 | 48 | 21'4641 | 8'7140 | $R = \frac{a\sqrt{5+2\sqrt{2}}}{2}$ | $\rho = \frac{a\sqrt{4+2\sqrt{2}}}{2}$ | $r = \frac{a(6+\sqrt{2})\sqrt{5+\sqrt{2}}}{17}$ |
| Gran Rombicuboctàedre | 26=12Q+8H+6 O | 48 | 72 | 61'7551 | 41'7990 | $R = \frac{a\sqrt{13+6\sqrt{2}}}{2}$ | $\rho = \frac{a\sqrt{12+6\sqrt{2}}}{2}$ | $r = \frac{3a(14+\sqrt{2})\sqrt{13+6\sqrt{2}}}{97}$ |
| Cub simus | 38=32T+6Q | 24 | 60 | 19'8564 | 7'8895 | | | |
| Icosidodecàedre | 32=20T + 12P | 30 | 60 | 29'3060 | 13'8355 | $R = \frac{a(1+\sqrt{5})}{2}$ | $\rho = \frac{a\sqrt{5+2\sqrt{5}}}{2}$ | $r = \frac{a(5+3\sqrt{5})}{8}$ |
| Dodecàedre truncat | 32=20T + 12 D | 60 | 90 | 100'9907 | 85'0396 | $R = \frac{a\sqrt{74+30\sqrt{5}}}{4}$ | $\rho = \frac{a(5+3\sqrt{5})}{4}$ | $r = \frac{5a(17\sqrt{2}+3\sqrt{10})\sqrt{37+15\sqrt{5}}}{488}$ |
| Icosàedre truncat | 32=12P + 20H | 60 | 90 | 72'6072 | 55'2877 | $R = \frac{a\sqrt{58+18\sqrt{5}}}{4}$ | $\rho = \frac{3a(1+\sqrt{5})}{4}$ | $r = \frac{9a(21+\sqrt{5})\sqrt{58+18\sqrt{5}}}{872}$ |
| Rombicosidodecàedre | 62=20T+12P+30Q | 60 | 120 | 59'3060 | 41'6153 | $R = \frac{a\sqrt{11+4\sqrt{5}}}{2}$ | $\rho = \frac{a\sqrt{10+4\sqrt{5}}}{2}$ | $r = \frac{a(15+2\sqrt{5})\sqrt{11+4\sqrt{5}}}{41}$ |
| Gran Rombicosidodecàedre | 62=30Q+20H+12 O | 120 | 180 | 174'2920 | 206'8034 | $R = \frac{a\sqrt{31+12\sqrt{5}}}{2}$ | $\rho = \frac{a\sqrt{31+12\sqrt{5}}}{2}$ | $r = \frac{a(105+6\sqrt{5})\sqrt{31+12\sqrt{5}}}{241}$ |
| Dodecàedre simus | 92=80T + 12P | 60 | 150 | 55'2867 | 37'6166 | | | |

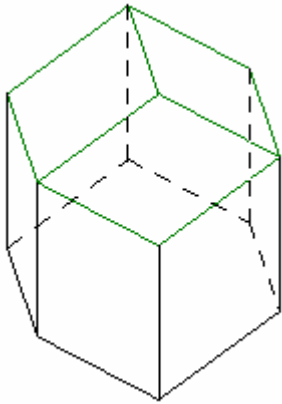
Propietat dels radis de les esferes circumscrita, inscrita i tangent a les arestes del políedre: $R \cdot r = \rho^2$

T triangles equilàters. Q quadrats. P pentàgons regulars. H hexàgons regulars. O octògons regulars. D decàgons regulars

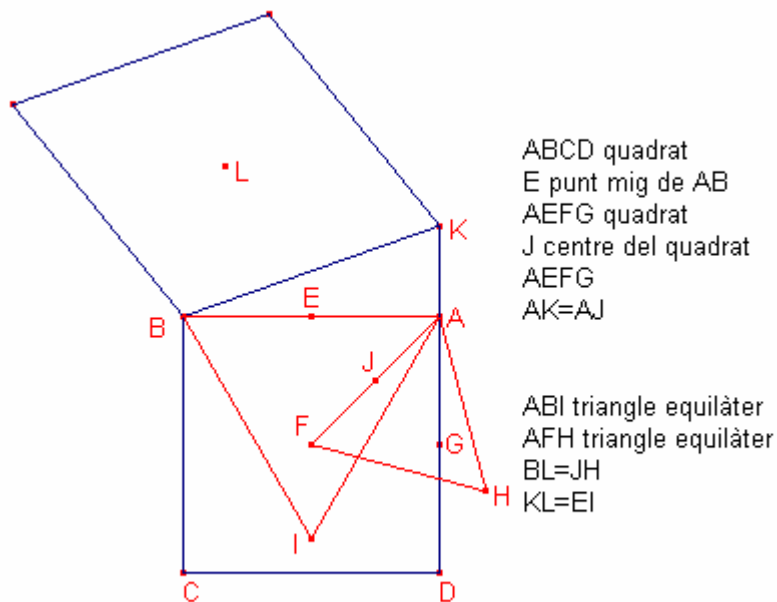
Nota: Les superfícies i els volums són aproximats, l'aresta és 1.

La bresca de mel

La bresca de mel està formada per un prisma hexagonal i un dodecàedre ròmbic.



Mòdul per construir la cara lateral i un rombe de la base:



Desenvolupament:

