

Considerem el sistema de referència afí
 $\{O; \{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}\}$

Sigueu els punts $A(a, 0, 0)$, $B(0, b, 0)$,
 $C(0, 0, c)$.

Sigueu les àrees: $P = S_{OAB}$, $Q = S_{OAC}$,

$R = S_{OBC}$, $S = S_{ABC}$.

Proveu que $P^2 + Q^2 + R^2 = S^2$.

Solució:

$\triangle OAB$, $\triangle OAC$, $\triangle OBC$ són triangles rectangles.

$$P = S_{OAB} = \frac{1}{2}|ab|.$$

$$Q = S_{OAC} = \frac{1}{2}|ac|.$$

$$R = S_{OBC} = \frac{1}{2}|bc|.$$

$$P^2 + Q^2 + R^2 = \frac{1}{4}((ab)^2 + (ac)^2 + (bc)^2).$$

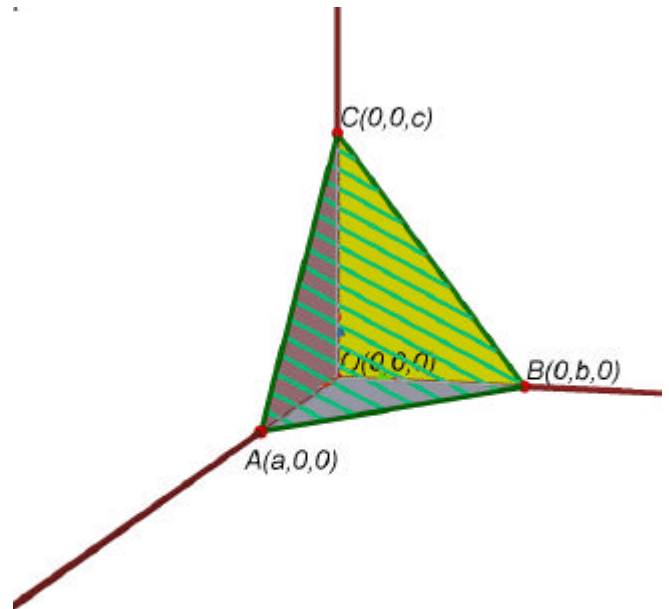
$$\vec{AB} = (-a, b, 0), \vec{AC} = (-a, 0, c).$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = (bc, ac, ab).$$

$$\|\vec{AB} \times \vec{AC}\| = \|(bc, ac, ab)\| = \sqrt{(bc)^2 + (ac)^2 + (ab)^2}.$$

$$S = S_{ABC} = \frac{1}{2}\|\vec{AB} \times \vec{AC}\| = \frac{1}{2}\sqrt{(ab)^2 + (ac)^2 + (bc)^2}.$$

$$S^2 = \frac{1}{4}((ab)^2 + (ac)^2 + (bc)^2).$$



$$S = \text{àrea}ABC = 10,6 \text{ cm}^2$$

$$S^2 = 112,97 \text{ cm}^4$$

$$P = \text{àrea}OAB = 5,3 \text{ cm}^2$$

$$Q = \text{àrea}OAC = 7,0 \text{ cm}^2$$

$$R = \text{àrea}OBC = 6,0 \text{ cm}^2$$

$$P^2 + Q^2 + R^2 = 112,97 \text{ cm}^4$$

