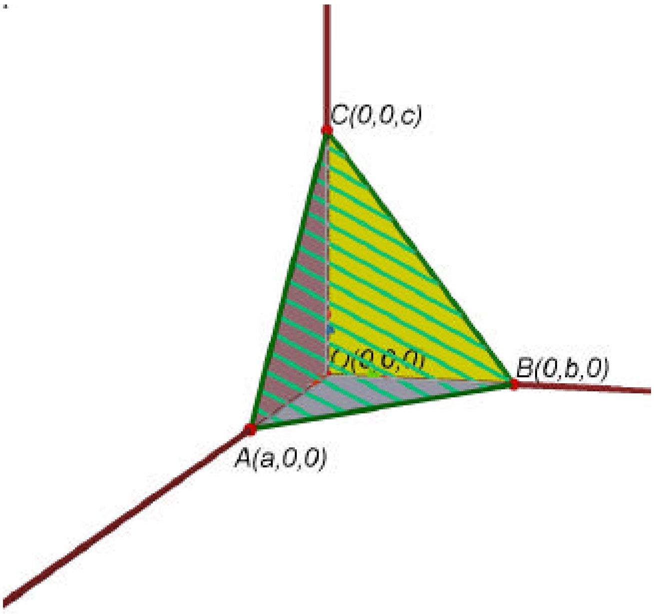
Considerem el sistema de referència afí
$\{O; \{e_1 = (1, 0, 0), e_2 = (0, 1, 0), e_3 = (0, 0, 1)\}\}$
Siguen els punts $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$.
Siguen les àrees: $P = S_{OAB}$, $Q = S_{OAC}$, $R = S_{OBC}$, $S = S_{ABC}$.
Proveu que $P^2 + Q^2 + R^2 = S^2$.



Solució:

$\overset{\Delta}{OAB}$, $\overset{\Delta}{OAC}$, $\overset{\Delta}{OBC}$ són triangles rectangles.

$$P = S_{OAB} = \frac{1}{2}|ab|.$$

$$Q = S_{OAC} = \frac{1}{2}|ac|.$$

$$R = S_{OBC} = \frac{1}{2}|bc|.$$

$$P^2 + Q^2 + R^2 = \frac{1}{4}\left((ab)^2 + (ac)^2 + (bc)^2\right).$$

$$\vec{AB} = (-a, b, 0), \ \vec{AC} = (-a, 0, c).$$

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} e_1 & e_2 & e_3 \\ -a & b & 0 \\ -a & 0 & c \end{vmatrix} = (bc, ac, ab).$$

$$\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\| = \|(bc, ac, ab)\| = \sqrt{(bc)^2 + (ac)^2 + (ab)^2}.$$

$$S = S_{ABC} = \frac{1}{2}\left\|\overrightarrow{AB} \times \overrightarrow{AC}\right\| = \frac{1}{2}\sqrt{(ab)^2 + (ac)^2 + (bc)^2}.$$

$$S^2 = \frac{1}{4}\left((ab)^2 + (ac)^2 + (bc)^2\right).$$